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A Straightforward hp-Adaptivity Strategy for Shock-Capturing with High-Order Discontinuous Galerkin Methods

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Abstract. In this paper, high-order Discontinuous Galerkin (DG) method is used to solve the two-dimensional Euler equations. A shock-capturing method based on the artificial viscosity technique is employed to handle physical discontinuities. Numerical tests show that the shocks can be captured within one element even on very coarse grids. The thickness of the shocks is dominated by the local mesh size and the local order of the basis functions. In order to obtain better shock resolution, a straightforward hp-adaptivity strategy is introduced, which is based on the high-order contribution calculated using hierarchical basis. Numerical results indicate that the hp-adaptivity method is easy to implement and better shock resolution can be obtained with smaller local mesh size and higher local order.

AMS subject classifications: 35L67, 65M60

Key words: hp-adaptivity, shock capturing, discontinuous Galerkin.

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1 Introduction

Discontinuous Galerkin (DG) methods [1–10] have recently become more and more popular for the solution of the Euler and Navier-Stokes equations of gas dynamics [1, 4, 5, 8, 10]. DG methods combine two important features which commonly characterize the finite element and finite volume methods. Firstly, high-order solutions can be obtained via using high-order polynomial approximation inside elements. Furthermore, upwinding can be easily implemented through using appropriate numerical fluxes over element interfaces. These two features make DG methods suitable for convection-dominated problems and highly accurate solutions can be obtained on relatively coarse grids if high-order scheme is used.

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However, for the cases involving strong discontinuities (for example, the shocks in the transonic flows and supersonic flows), the unlimited high-order solution oscillates near the discontinuities. There are many methods that can be used to eliminate the numerical oscillations. The most straightforward one is to reduce the order of the polynomial near the shocks. However, the accuracy of the scheme can be seriously degraded. Hence, the mesh-refinement has to be used to provide a satisfactory solution. The weighted essentially non-oscillatory (WENO) schemes can also be used since they provide stable discretization near discontinuities and still maintain high-order accuracy. The main drawback is that they become to be highly consuming when the degree of the approximating polynomial is increased. There are also some other methods based on reconstructing the oscillatory solutions computed using a high order method. But it is still an issue to extend these methods to multiple dimensions. Recently, an artificial viscosity method (sub-cell shock-capturing method) was introduced to capture the shocks when high-order DG is used [10]. In this method, the accuracy of the solution in the neighborhood of the shock becomes $\mathcal{O}(h/p)$ and the shock profile can be resolved in one element when high order is used.

In this paper, the high-order DG methods combined with the sub-cell shock-capturing technique are used to simulate compressible flows with shocks. The diffusion term introduced by the artificial viscosity technique is discretized using LDG [2] scheme as shown in [10]. Newton method [4] is employed to solve the nonlinear discrete systems. For each nonlinear iteration, the resultant sparse linear system is solved using a block-Gauss Seidel method. Since Newton method is relatively sensitive to the initial guess, a hierarchical solution procedure is suggested in this paper.

Numerical results indicate that the shocks can be well resolved inside one element when the above artificial viscosity technique is used. However, the shock thickness is dominated by the local order and the local mesh size. In order to reach the required shock thickness, an *hp*-adaptivity method is introduced in this work. Firstly, a smoothness sensor based on the high-order contribution is used to detect the position of the shocks. Secondly, the local order of the basis is enhanced to improve the resolution. If the solution near the shocks is still not accurate enough when the local order reaches 4, the local mesh will be refined. Numerical results show that the straightforward *hp*-adaptivity strategy can produce reasonable mesh structure and order distribution which can provide accurate shock resolution at relatively low expense.

2 Governing equations

The two-dimensional Euler equations in conservative form can be written as

$$\frac{\partial \mathbf{U}}{\partial t}+\nabla\cdot\mathbf{F}(\mathbf{U})=0, \tag{2.1}$$