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A Posteriori Error Estimates of Triangular Mixed Finite Element Methods for Semilinear Optimal Control Problems

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Abstract. In this paper, we present an a posteriori error estimates of semilinear quadratic constrained optimal control problems using triangular mixed finite element methods. The state and co-state are approximated by the order $k \leq 1$ Raviart-Thomas mixed finite element spaces and the control is approximated by piecewise constant element. We derive a posteriori error estimates for the coupled state and control approximations. A numerical example is presented in confirmation of the theory.

AMS subject classifications: 49J20, 65N30

Key words: Semilinear optimal control problems; mixed finite element methods; a posteriori error estimates.

1 Introduction

Optimal control problems have attracted substantial interest in recent years due to their applications in aero-hydrodynamics, combustion, exploration and extraction of oil and gas resources, and engineering. The past decade has seen significant developments in theoretical and computational methods for optimal control problems. The finite element method is a valid numerical method of studying the partial differential equation, but it is not deeply studied in solving optimal control problems. For optimal control problems governed by linear elliptic equations, there are some pioneering

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work on numerical approximation by Falk [9] and Mossino [21]. An optimal control problem for a two-dimensional elliptic equation is investigated with pointwise control constraints in Meyer and Rösch [19]. A systematic introduction of the finite element method for optimal control problems can be found in, for instance, [12, 13, 16] and the references cited therein. Most of these researches have been, however, only for the standard finite element methods for optimal control problems.

In many optimal control problems, the objective functional contains the gradient of the state variables. Thus, the accuracy of the gradient is very important in the numerical discretization of the state equations. Mixed finite element methods are appropriate for the state equations in such cases since both the scalar variable and its flux variable can be approximated to the same accuracy by using such methods. In [5, 23] the authors presented a priori error estimates and superconvergence of mixed finite element methods for linear optimal control problems. However, there does not seem to exist much work on theoretical estimates of mixed finite element methods for nonlinear optimal control problems.

Adaptive finite element approximation is a most important means to boost accuracy and efficiency of the finite element discretization. Adaptive finite element approximation uses a posteriori error indicator to guide the mesh refinement procedure. In [25], the author proposed a posteriori error estimates of gradient recovery type for linear optimal control problems. Liu and Yan investigated a posteriori error estimates and adaptive finite element approximation for optimal control problems governed by Stokes equations in [18]. In [3, 4, 24], we derived a priori error estimates and superconvergence for linear quadratic optimal control problems using mixed finite element methods. A posteriori error estimates of mixed finite element methods for general convex optimal control problems was addressed in [6–8].

The purpose of this work is to obtain a posteriori error estimates of triangular mixed finite element methods for quadratic optimal control problems governed by semilinear elliptic equations. Compared with the related work [11], the present paper gives the first a posteriori error estimate for semilinear quadratic optimal control problems when they are discretized by Raviart-Thomas mixed finite element methods.

In this paper, we consider the following quadratic optimal control problems governed by semilinear elliptic equations:

$$\min_{u\in K\subset U}\left\{\frac{1}{2}\parallel \boldsymbol{p}-\boldsymbol{p}_d\parallel^2+\frac{1}{2}\parallel y-y_d\parallel^2+\frac{\nu}{2}\parallel u\parallel^2\right\}, \tag{1.1}$$

$$\mathrm{div}\boldsymbol{p}+\phi(y)=u, \qquad \text{in } \Omega, \tag{1.2}$$

$$\boldsymbol{p}=-A\nabla y, \qquad \text{in } \Omega, \tag{1.3}$$

$$y=0, \qquad \text{on } \partial\Omega, \tag{1.4}$$

where the bounded open set $\Omega\subset\mathbb{R}^2$, is a convex polygon with boundary $\partial\Omega$, $f\in L^2(\Omega)$, and K is a closed convex set in $L^2(\Omega)$. For any $R>0$ the function $\phi(\cdot)\in W^{2,\infty}(-R,R)$, $\phi'(y)\in L^2(\Omega)$ for any $y\in H^1(\Omega)$, and $\phi'(y)\geq 0$. Furthermore, we assume the coefficient matrix

$$A(x)=(a_{i,j}(x))_{2\times 2}\in(W^{1,\infty}(\Omega))^{2\times 2},$$