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A Modified Nonconforming 5-Node Quadrilateral Transition Finite Element

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Abstract. This paper analyzes a nonconforming 5-node quadrilateral transition finite element for Poisson equation. This element was originally proposed by Choi and Park [Computers and Structures, 32 (1989), pp. 295–304 and Thin-Walled Structures, 28 (1997), pp. 1–20] for the analysis of Mindlin plates. We show the consistency error of this element is only $\mathcal{O}(h^{1/2})$ over the transition edges of the quadrilateral subdivision. By modifying the shape functions with respect to mid-nodes, we get an improved version of the element for which the consistency error is $\mathcal{O}(h)$. Numerical examples are provided to verify the theoretical results.

AMS subject classifications: 65N12, 65N30

Key words: Nonconforming finite element, transition element, consistency error.

1 Introduction

Adaptive methods for the numerical solution of the PDEs are now standard tools in science and engineering to achieve better accuracy with minimum degrees of freedom. In the adaptive analysis, the mesh is locally refined according to the estimated error distribution through repeating the working loop comprised of finite element analysis, error estimation, element or edge marking and mesh refinement until the error decreases to a prescribed level.

As far as adaptive quadrilateral mesh refinement is concerned, when a 4-node quadrilateral element is subdivided into smaller elements, new nodes (hanging nodes) appear on the boundaries of its immediate neighborhoods, which are known as transition elements. In these elements, each edge may possess a mid-side node and the edge is shared by the adjacent bilinear elements. When the conventional quadratic interpolation is used along the 3-node edge, interelement compatibility is violated. There are several ways to secure this [8, 16].

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The first way is to constrain the mid-side node displacement of the transition element to be the average of the displacement at the two corner nodes of the same edge (Fig. 1(a): $u_c=(u_a+u_b)/2$) [19, 20]. However, this nullifies the accuracy enhancement effect of the mid-side nodes and constraint equations are computationally inefficient [1].

The second way is to use a meshing technique of finite element layout shown in Fig. 1(b). In this case, no constraints are imposed, but the use of distorted elements is inevitable.

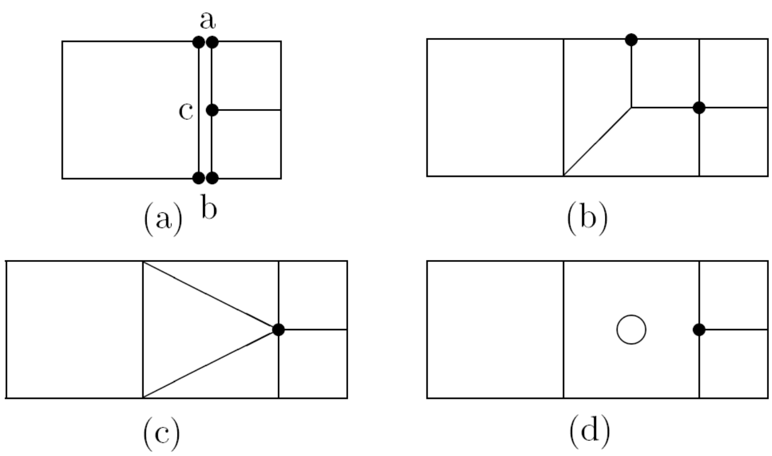

Figure 1: Examples of mesh transition.

The third way is to use macro-elements formed by compatible 4-node bilinear quadrilaterals and compatible 3-node linear triangles (Fig. 1(c)). Since the triangular elements produce worse results than the quadrilaterals in general, the introduction of triangular elements in the mesh may cause poor solutions even if the quadrilaterals in the mesh behave well.

The fourth way is to introduce transition elements (Fig. 1(d)) to connect directly the different layer patterns. Well-established transition element can overcome some of the aforementioned meshing problems. Gupta [13] derived a set of compatible interpolation functions for the quadrilateral transition elements. The displacement interpolation along a 3-node element edge is continuous piecewise bilinear instead of quadratic, thus preserve the interelement compatibility. Carstensen and Hu [5] provided a method to preseve the interelement compatibility with just modifying the nodal basis functions of the immediate neighborhoods of the hanging nodes. McDill [17] and Morton [18] presented the 3D counterpart of Gupta's conforming transition elements. Choi et al. [6–9] proposed a set of 2D and 3D nonconforming transition elements. Wan et al. [16,21], Wu et al. [22], Duan et al. [11], Hasan et al. [14] and Peters et al. [19] constructed some hybrid stress and enhanced strain transition quadrilateral/hexahedral elements.

In this paper, we shall analyze the nonconforming 5-node quadrilateral transition