Advances in Applied Mathematics and Mechanics
Adv. Appl. Math. Mech., Vol. **2**, No. 6, pp. 763-783

DOI: 10.4208/aamm.09-m0997
December 2010

Numerical Study of Double Diffusive Convection in Partially Heated Vertical Open Ended Cylindrical Annulus

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Received 29 October 2009; Accepted (in revised version) 8 July 2010
Available online 26 August 2010

Abstract. A numerical investigation of laminar natural double diffusive convection in an open ended vertical cylindrical annulus with unheated entry and unheated exit is performed. Both boundary conditions of uniform wall temperature/uniform wall concentration (UWT/UWC) and uniform heat flux/uniform mass flux (UHF/UMF) are considered. Results of dimensionless induced volume rate Q, average Nusselt number $\overline{Nu}$ and Sherwood number $\overline{Sh}$ are obtained for air flow under various buoyancy ratio N, Grashof numbers due to heat and mass transfer Gr_T and Gr_M, Schmidt number Sc and combinations of unheated entry, heated section and unheated exit length. Since the flow under consideration is a boundary layer type, the governing partial differential equations was discretized to a linear system of equations by the use of an implicit finite difference method. The nonlinear convective terms are approximated by second upwind difference method for the numerical stability. The numerical results reveal that the presence of unheated entry and unheated exit severely affects the heat and mass transfer rates. The numerical solutions are found to approach asymptotically the closed form solutions for fully developed flow. Further, the present numerical results are validated with the existing solutions for pure thermal convection and are found to be in good agreement.

AMS subject classifications: 65L12, 76D05, 80A20

Key words: Discrete heating, cylindrical annulus, upwind difference, doubles diffusive convection, buoyancy ratio.

1 Introduction

Many practical systems involve convective heat and mass transfer in fluids in heated, vertical, open-ended channels like circular tubes, parallel plates and annular cavities.

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Noticeable examples include the chemical distillatory processes, design of heat exchangers, channel type solar energy collectors and thermo-protection systems. Hence, the characteristics of natural convection heat and mass transfer are relatively important in the above mentioned applications. Among the above geometries, vertical open-ended cylindrical annulus whose side walls are at different temperatures and concentrations is the most general cavity since it includes the circular tubes and parallel plates as its limiting geometries. Such systems are of practical importance in the field of double pipe arrangements, particularly the fuel elements of nuclear reactors during the shut-off periods. Also the present day technological and engineering environment and style of living in metropolis are demanding hazard-free and safe electrical power supply equipments. In addition, to provide a cleaner and risk-free environment, considerable emphasis has been given on the pollution free equipments.

The natural convection heat transfer in vertical open annular duct flows induced by the thermal buoyancy alone has been investigated in great detail in the literature. El-Shaarawi and Sarhan [1] presented numerical results for the laminar natural convection heat transfer in an open ended vertical concentric annulus of radius ratio 0.5 with one wall being isothermal and other wall being adiabatic. Coney and El-Shaarawi [2] used the finite difference analysis for the incompressible laminar convective flow in concentric annuli with simultaneous development of hydrodynamic and thermal boundary layers and found that the rate of heat transfer for the case of isothermal inner wall and adiabatic outer wall is higher than that of the case of isothermal outer wall and adiabatic inner wall. Later, El-Shaarawi and Sarhan [3] developed a finite difference scheme for the laminar free convective flow in an open ended vertical concentric annuli with rotating inner wall and concluded that heating the inner cylinder has always stabilising effect while heating the outer cylinder has either destabilising or stabilising effect, depending on the nature of the rotation. Numerical investigation of laminar mixed convection in a vertical annulus was studied by Hashimoto et al. [4]. Al-Nimar [5] obtained an analytical solution for transient laminar fully developed free convection in vertical concentric annulus corresponding to four thermal boundary conditions. Recently, natural convection heat and mass transfer in vertical concentric annuli with film evaporation and condensation is performed numerically by Yan and Lin [6].

Natural convection due to heat and/or mass transfer in vertical parallel plates is also studied in detail. A numerical solution based on a finite difference scheme was first obtained by Bodia and Osterle [7] for the development of free convection heat transfer between heated vertical parallel plates. Their finite difference calculations show that the development height is rather significant and for most situations the assumption of fully developed flow is not valid. Yan and Lin [8] considered the effect of the discrete heating on the vertical parallel channel flows driven by buoyancy force and found that the rate of heat transfer is more for continuous heating. The effect of unheated entry and unheated exit section on the natural convection of air flow in a vertical parallel plate channel is numerically investigated by Lee [9]. He considered the uniform heat flux (UHF) and uniform wall temperature (UWT) thermal bound-