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Numerical Solution of Fractional Partial Differential Equations by Discrete Adomian Decomposition Method

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Abstract. In this paper we find the solution of linear as well as nonlinear fractional partial differential equations using discrete Adomian decomposition method. Here we develop the discrete Adomian decomposition method to find the solution of fractional discrete diffusion equation, nonlinear fractional discrete Schrodinger equation, fractional discrete Ablowitz-Ladik equation and nonlinear fractional discrete Burger's equation. The obtained solution is verified by comparison with exact solution when $\alpha = 1$.

AMS subject classifications: 35R11

Key words: Discrete Adomian decomposition method, Caputo fractional derivative, fractional discrete Schrodinger equation, fractional discrete Burger's equation.

1 Introduction

Fractional differential equations have been the focus on many studies due to their frequent appearance in various fields such as physics, chemistry and engineering. The fractional derivative has been occurring in many physical problems such as frequency dependent damping behavior of materials, motion of a large thin plate in Newtonian fluid, creep and relaxation functions for viscoelastic material, the $PI^{\lambda}D^{\mu}$ controller for the control of dynamical systems etc. Phenomena in electromagnetic, acoustics, viscoelasticity, electrochemistry and material science are also described by differential equations of fractional order [30]. The applicability of this type of equations motivates us to construct efficient methods for solving fractional differential equations. The popular among them are integral transform method [30, 32], iterative method [13, 19], and Adomian decomposition method [4, 5, 17].

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Adomian decomposition method is introduced by Adomian and [4,5] has been proven a very useful tool to deal with nonlinear equations. Wazwaz [39] has applied Adomian decomposition method to solve variety of differential equations. While Shawagfeh [34] has employed Adomian decomposition method for solving nonlinear fractional differential equations, Daftardar-Gejji and Jafri have obtained solution of numerous problems [12,25] by using Adomian decomposition method. Also Dhaigude and Birajdar [18] extended the discrete Adomian decomposition method for obtaining the numerical solution of system of fractional partial differential equations.

Recently fractional diffusion equations have attracted attention of many researchers due to its wide applicability both in the theory of mathematical science and technology. They have been used in modelling many physical and chemical processes, heat, mass or electron transfer, pollutants or liquid transport through porous media and engineering problems. Fractional diffusion equations account for typical anomalous feature which is observed in many systems, e.g., the dispersive transports in amorphous semiconductors, porous medium, colloid, proteins, biosystems or even in ecosystems [21, 22, 24, 29]. The recent papers [14, 23, 31] on fractional diffusion equations are valuable in this field. Wyss [38] considered the time fractional diffusion equation and the solution is given in closed form in terms of Fox function. Schneider and Wyss [33], Dhaigude and Nikam [20] considered the time fractional diffusion equation and wave equation and obtained their solutions. Existence and uniqueness of solution of fractional diffusion equations are well studied by Dhaigude [16]. We develop the discrete Adomian decomposition method for fractional discrete diffusion equation.

The nonlinear Schrodinger equation for integer order [15, 27] is a typical dispersive nonlinear partial differential equation that plays a key role in a variety of areas in mathematical physics. Wazwaz employed Adomian decomposition method for solving different types of integer order Schrodinger equation in [36, 37]. Kaya and El-Sayed [26] have solved coupled Schrodinger-KdV equation for integer order using Adomian decomposition method. The Ablowitz-Ladik equation is discovered and studied by Ablowitz-Ladik in [1, 2]. It is the particular case of Schrodinger equation the nonlinear term in the Schrodinger equation is replaced by space discrete form. Bratsos et al. [9] proposed the discrete Adomian decomposition method for the solution of integer order Schrodinger equation. We extend discrete Adomian decomposition method to obtain the numerical solution of nonlinear fractional discrete Schrodinger equation.

In 1915, Bateman [7] studied the Burger's equation which was introduced by Burger [10] in a mathematical modeling of turbulence, and hence it is referred as "one dimensional Burger's equation" which has applications in physics, fluid dynamics, gas dynamics, heat conduction etc. This equation arises in the theory of shock waves, in turbulence problems and in continuous stochastic processes, gas dynamics, heat conduction, elasticity [6, 8]. Abbasbandy and Darvishi [3] developed the Adomian decomposition method for numerical solution of Burger's equation for integer order. Zhu et al. [40] obtained the solution of two dimension integer order Burger's equation by using discrete Adomian decomposition method. We develop the discrete Adomian decompo-