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Two-Level Newton Iteration Methods for Navier-Stokes Type Variational Inequality Problem

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Abstract. This paper deals with the two-level Newton iteration method based on the pressure projection stabilized finite element approximation to solve the numerical solution of the Navier-Stokes type variational inequality problem. We solve a small Navier-Stokes problem on the coarse mesh with mesh size H and solve a large linearized Navier-Stokes problem on the fine mesh with mesh size h. The error estimates derived show that if we choose $h=\mathcal{O}(|\log h|^{1/2}H^3)$, then the two-level method we provide has the same H^1 and L^2 convergence orders of the velocity and the pressure as the one-level stabilized method. However, the L^2 convergence order of the velocity is not consistent with that of one-level stabilized method. Finally, we give the numerical results to support the theoretical analysis.

AMS subject classifications: 65N30

Key words: Navier-Stokes equations, nonlinear slip boundary conditions, variational inequality problem, stabilized finite element, two-level methods.

1 Introduction

In this paper, we deal with the steady Navier-Stokes equations:

$$\begin{cases} -\mu\Delta u+(u\cdot\nabla)u+\nabla p=f, & \text{in } \Omega, \\ \text{div} u=0, & \text{in } \Omega, \end{cases} \tag{1.1}$$

where $\Omega\subset\mathbb{R}^2$ is a bounded and convex domain. $\mu>0$ denotes the kinetic viscous coefficient, u and p denote the velocity and the pressure, respectively. f denotes the external

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body force. $\mathrm{div}u=0$ implies that the fluid is incompressible. We suppose that the boundary $\partial\Omega$ of Ω is composed of two parts Γ and S which satisfy meas$(\Gamma)\neq 0$, meas$(S)\neq 0$, $\Gamma\cap S=\emptyset$, $\overline{\Gamma\cup S}=\partial\Omega$. Unlike the usual whole Dirichlet boundary conditions, we consider the following the nonlinear slip boundary conditions of friction type:

$$\begin{cases} u=0, & \text{on } \Gamma, \\ u_n=0, \quad -\sigma_\tau(u)\in g\partial|u_\tau|, & \text{on } S, \end{cases} \tag{1.2}$$

where $g\geq 0$ is a scalar function. $u_n=u\cdot n$ and $u_\tau=u\cdot\tau$ are the normal and tangential components of the velocity, where n and τ stand for the unit vector of the external normal and the tangential vector to S. $\sigma_\tau(u)=\sigma\cdot\tau$, independent of p, is the tangential components of the stress vector σ defined by $\sigma_i=\sigma_i(u,p)=(\mu e_{ij}(u)-p\delta_{ij})n_j$, where $e_{ij}(u)=\partial u_i/\partial x^j+\partial u_j/\partial x^i$, $i,j=1,2$. The set $\partial|u_\tau|$ denotes a subdifferential of the absolute value function at the point u_τ, which is defined by

$$\partial|u_\tau|=\big\{b\in\mathbb{R}:|h|-|u_\tau|\geq b\cdot(h-u_\tau),\ \forall\, h\in\mathbb{R}\big\}.$$

The Navier-Stokes equations with nonlinear slip boundary conditions of friction type is firstly introduced by Fujita in [1] and appears in the modeling of blood flow in a vein of an arterial sclerosis patient. There have some theoretical results, especially for the well-posedness analysis of the Stokes problem. We refer to Fujita [2–4], Saito [5], Li [6] and the references cited therein. Some scholars have focused on the numerical methods. For example, Suito and his collaborates have applied the boundary conditions (1.2) to some flow phenomena by the finite difference methods in [7–9], such as the oil flow over or beneath sand layers and the blood flow in the thoracic aorta. Ayadi and his collaborates in [10] studied the finite element approximation for the Stokes problem, where they use the P_1b-P_1 element and derived the error estimates in virtue of the Lagrange multiplier method. Kwshiwabara in [11] used the Taylor-Hood element and obtained the optimal error estimates for the Stokes problem. Recently, we in [12] applied the pressure projection stabilized finite element method to the steady Navier-Stokes problem and constructed the simple and the Oseen two-level iteration schemes. We showed that if the coarse mesh size H and the fine mesh size h satisfy $h=\mathcal{O}(H^2)$, then the error estimates indicate the simple or Oseen two-level methods will provide the same order of the approximation as the usual one-level stabilized finite element method [13]. Much research works have been done about the finite element analysis the variational inequality problems associated with the Navier-Stokes equations. We refer to the following works [14–16] and the references cited therein.

In this paper, based on the Newton iteration scheme [17–19], we continue to study the two-level finite element methods for the Navier-Stokes equations with the boundary conditions (1.2). The main idea is solving a small Navier-Stokes type variational inequality problem on the coarse mesh with mesh size H and solving a large linearized Navier-Stokes type variational inequality problem on the fine mesh with mesh size h in virtue