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Steady States of Sheared Active Nematics

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Abstract. A continuum hydrodynamic model has been used to characterize flowing active nematics. The behavior of such a system subjected to a weak steady shear is analyzed. We explore the director structures and flow behaviors of the system in flow-aligning and flow tumbling regimes. Combining asymptotic analysis and numerical simulations, we extend previous studies to give a complete characterization of the steady states for both contractile and extensile particles in flow-aligning and flow-tumbling regimes. Another key prediction of this work is the role of the system size on the steady states of an active nematic system: if the system size is small, the velocity and the director angle files for both flow-tumbling contractile and extensile systems are similar to those of passive nematics; if the system is big, the velocity and the director angle files for flow-aligning contractile systems and tumbling extensile systems are akin to sheared passive cholesterics while they are oscillatory for flow-aligning extensile and tumbling contractile systems.

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1 Introduction

Active liquid crystals are examples of active complex fluids receiving increasing theoretical and experimental attention. Such materials are called active because they continuously burn energy, for example, in the form of adenosine triphosphate (ATP), and this drives them out of thermodynamic equilibrium even when there is no external force. This is in sharp contrast to the physics of passive soft-matter systems where non-equilibrium

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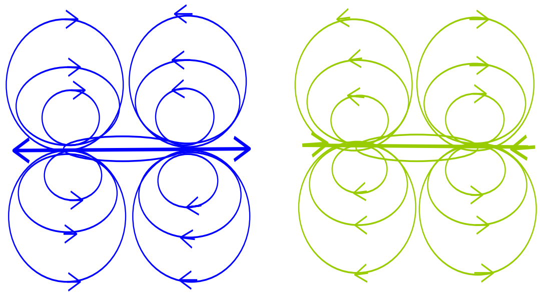

Figure 1: The dipolar flow fields surrounding a extensile (left) $(\delta<0)$ and a contractile (right) $(\delta>0)$ particle. The vertical arrows represent the director field, which is along the rod axis for rodlike swimmers.

behavior typically occurs only in the presence of an external driving force (such as shear). These systems are common in nature such as acto-myosin filaments, cytoskeletal gels interacting with motors, bacterial colonies, and swarming fishes [12, 13], but also occur in technological applications, as engineers have tried to design artificial swimmers to perform various functions [7, 11]. They are also interesting from a more fundamental perspective as their dynamic phenomenon such as bioconvection [6] and the spontaneous flow [8,17] are both physically fascinating and potentially of great biological significance.

Conventional liquid crystals exhibit a rich dynamic behavior when subject to external forcing, such as shear or applied magnetic and electric fields. This includes phase transitions, shear banding [16], and even the turbulent and chaotic behavior in the presence of shear [2]. Activity imparts non-trivial physical properties to an active system and leads to its peculiar phenomena such as bacterial swarming [10], bioconvection phenomenon or the so-called weak turbulence [6] and the spontaneous flow in the absence of externally applied forces, both stationary and oscillatory [8,9,14,17], in sharp contrast to their passive counterparts.

Active particles exert forces on the surrounding fluid, resulting in local extensile or contractile stresses proportional to the amount of orientational tensor: $\tau^a=\delta\mathbf{nn}$, where $\mathbf{n}$ is the orientation director in the nematic phase which is generally described by a unit vector, and δ is the amplitude of the dipolar forces exerted by the particle to swim. The sign of δ determines whether the dipolar flow field generated by the swimming suspension is extensile $(\delta<0)$ or contractile $(\delta>0)$, as illustrated in Fig. 1. In the swimmer literature, the former situation describes "pushers", i.e., most bacteria including E. Coli and Bacillus Subtilis, while the latter corresponds to "pullers" including Chlamydomonas. We note that this distinction is by no means exclusive, as certain highly symmetric organisms such as spherical multicellular algae (e.g., Volvox) may fall between the extensile-contractile or pusher-puller distinction.

A striking property of active liquid crystal films is the onset of spontaneous flow above a critical film thickness first identified by Voituriez et al. [17]. Marenduzzo et al. [14,15] have numerically studied the active nematic hydrodynamics in both 1D and 2D geometry based on the conformation tensor theory of Beris and Edwards [1]. Edwards et