Advances in Applied Mathematics and Mechanics
Adv. Appl. Math. Mech., Vol. **16**, No. 4, pp. 980-1008

DOI: 10.4208/aamm.OA-2022-0321
August 2024

High-Resolution and High-Precision Weighted Essentially Non-Oscillatory Scheme for Compressible Flow Simulations

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Received 5 December 2022; Accepted (in revised version) 6 June 2023

Abstract. In this paper, a new high-resolution WENO-MIM scheme that can achieve optimal accuracy at high-order critical points is developed. The nonlinear weight function of the new scheme can be obtained by adding a freely adjustable term with a parameter λ to the mapping function of the WENO-IM scheme. A sufficient condition shows the shock-capturing ability of WENO-MIM will be enhanced with the increase of λ. The parameter $\lambda = 467$ obtained from experience can guarantee the new scheme achieves high resolution. Numerical example results show that the present scheme can achieve optimal accuracy at high-order critical points and perform significantly better than other WENO schemes in highly efficient computing of various compressible fluid problems.

AMS subject classifications: 65M06, 35L65

Key words: WENO scheme, high resolution, optimal accuracy, mapping function, freely adjustable term.

1 Introduction

The WENO scheme is mainly used for numerical simulation of flow problems involving discontinuities and finely smooth structures, which can maintain ENO properties with fine shock-capturing properties and achieve high-order accuracy. Since Liu et al. [1] developed the WENO scheme by improving the ENO method designed by Harten et al. [2], the WENO scheme has been widely used in fluid mechanics, and aerospace because of its high precision and essentially no-oscillatory. Jiang and Shu [3] proposed a technique that can measure the smoothness of sub-stencils and presented a framework

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that can build various higher-order WENO methods, called WENO-JS. Since then, researchers have conducted a series of tests, evaluations, and improvements to the WENO method [4–7].

Borges et al. [8] developed a WENO-Z method that achieves optimal orders at or near critical points by constructing a global smoothness indicator. Since then, researchers have proposed several new global smoothness indicators to improve the numerical performance of the WENO-Z type scheme. Castro et al. [9] developed a general higher-order WENO-Z by incorporating Taylor expansions of sub-stencils on the WENO scheme. Ha et al. [10] proposed a new smoothness indicator that evaluates the local smoothness of a function within a stencil and developed a new WENO-Z scheme, WENO-NS. Acker et al. [11] found that the resolution of the WENO scheme could be improved by increasing the weights of less smooth sub-stencils and developing the high-resolution WENO-Z+ scheme. Wang et al. [12] found that the accuracy of the fifth-order WENO-Z decreases at high-order critical points. They added a modifier function to the weight function of WENO-Z and developed a high-accuracy WENO-D. In addition, many scholars have used various methods to improve the global smoothing factor and developed many WENO-Z-type schemes [13–16].

Theory and numerical results show that the accuracy of the fifth-order WENO-JS method decreases to the third-order at or near the critical point in the smooth region. Henrick et al. [17] constructed a mapping function to improve the accuracy of the weight function of WENO-JS and developed a WENO-M scheme. Feng et al. [18] found that the mapping function of WENO-M produces more errors near discontinuity points, and the solutions of the seventh-order WENO-M method are not as accurate as those of WENO-JS in long-term simulations. They introduced a new piecewise continuous mapping function to overcome this potential loss of accuracy and developed a new mapped WENO method, WENO-PM6. Later, Feng et al. [19] developed a two-parameter mapping function and a new higher-order mapped WENO method, WENO-IMS, and the new method with $n=2$, $A=0.1$ can achieve optimal accuracy at the first-order critical point. Wang et al. [20] developed a family of smooth mapping functions for three-parameter rational functions and specified a new higher-order mapped WENO method, WENO-RM (m, n, k). The WENO-RM method with $(m, n, k)=(2, 6, 0)$ can achieve optimal accuracy at the first-order critical point. Vevek et al. [21] introduced an adaptive parameter in the mapping function of WENO-IM and developed a seventh-order adaptive WENO method (WENO-AIM) that achieves the best accuracy at or near the second-order critical point. To reduce the cost of the mapping process of the WENO-M method, Hong et al. [22] designed a pre-propagation mapping method independent of the mapping function. And they developed a WENO-M scheme without a mapping function. Different from the above, which directly maps the weight function of WENO-JS to reconstruct a new nonlinear weight function, Hu et al. [23] developed a mapping function for the smoothness indicators and designed a new mapped WENO scheme, WENO-IM. The WENO-IM can achieve optimal accuracy at high-order critical points, and its resolution is weaker than WENO-M and WENO-Z. In addition, many scholars have designed various mapping