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Partial Topology Identification of Stochastic Multi-Weighted Complex Networks Based on Graph-Theoretic Method and Adaptive Synchronization

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Abstract. This article aims to identify the partial topological structures of delayed complex network. Based on the drive-response concept, a more universal model, which includes nonlinear couplings, stochastic perturbations and multi-weights, is considered into drive-response networks. Different from previous methods, we obtain identification criteria by combining graph-theoretic method and adaptive synchronization. After that, the partial topological structures of stochastic multi-weighted complex networks with or without time delays can be identified successfully. Moreover, response network can reach synchronization with drive network. Ultimately, the effectiveness of the proposed theoretical results is validated through numerical simulations.

AMS subject classifications: 60H10, 93D05, 93E12

Key words: Partial topology identification, graph-theoretic method, multi-weighted complex networks, adaptive pinning control, nonlinear coupling.

1 Introduction

Complex networks, including industrial networks, financial networks, transportation networks and neural networks, are penetrated into almost all aspects of real world [1–7]. Till now, the investigation of complex networks has been extremely extensive. In addition to synchronization and stability [8–17], the research of topology identification has attached special attention [18–23].

In the literature, topological structures of many complex networks are assumed to be known [24–26]. However, the reality is that only a small section of topological structures are known or even completely unknown. Particularly, topological structures will constantly change with increasing and decreasing of network vertices or arcs. Therefore, it is practical to figure out the unknown topological structures of complex networks.

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In [18–23], they identified the whole topological structures of complex network. However, in many practical networks, only partial topological structures are what we need. For instance, in a social network, if the message is only linked to work, we only need to transmit it to colleagues in the list of friends. Similarly, when searching articles in Web of Science, we may only want to know articles in the same research field. Considering the circumstances mentioned above, it will result in high control cost if we identify the whole topological structures. To reduce the control cost, pinning control is a powerful technique since only a part of vertices can receive the control input directly. Therefore, it is essential and indispensable to address the problem of partial topology identification with pinning control.

Existing research recognises the critical role played by partial topology identification. In [27–30], the authors studied the partial topology identification of complex dynamical networks via a pinning mechanism. However, the above literatures [27–30] focus on partial topology identification of single weighted complex networks. Nevertheless, many real world networks can be modelled as coupled systems with multi-weights. They have different coupling forms among vertices. Examples of this kind of multi-weighted networks are ubiquitous. For instance, a social network includes the relationship among friends, relatives and colleagues. A traffic network includes the transportation among cars, planes and bikes. As we all know, in recent years, though some articles contribute to researching the whole topological structures of multi-weighted complex network [20, 31], there are only few results about partial topology identification of multi-weighted complex networks [32]. However, in [32], the model is linearly coupled. Therefore, it is of great significance to research partial topology identification of more general multi-weighted complex networks. Furthermore, in [27–31], the models considered have been largely restricted to deterministic ordinary differential equations. In fact, multi-weighted complex networks are inevitably affected by various types of environmental noise [33–37]. However, it should be noted that there are few papers about partial topology identification [32] of stochastic multi-weighted complex networks. Hence, to fill the gap, this paper attempts to identify partial topological structures of multi-weighted complex networks with stochastic disturbance and nonlinear couplings.

For the previous research of topology identification, researchers usually propose identification criteria based on Lyapunov method. Although this method is quite classical and widespread, it is still a challenge to construct an appropriate global Lyapunov function directly. In recent years, many scholars used graph theory to construct global Lyapunov function indirectly and further investigated stability and boundedness of complex networks [38–44]. This method uses some results about graph theory, so it is always called graph-theoretic method. It is an effective technique to systematically construct a global Lyapunov function by using the weighted summation of vertex Lyapunov functions. Till now, the new graph-theoretic method has rarely been applied on the partial topology identification of stochastic multi-weighted complex networks with nonlinear couplings.

Motivated by aforementioned discussions, this paper attempts to use graph-theoretic method to study partial topology identification of stochastic multi-weighted complex