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A Two-Dimensional Thermoelastic Analysis of a Cylindrical Shell Made of FGM with Temperature-Dependent Physical Properties

Hamed Panahi-Kalus, Amin Moosaie and Mehrdad Ahmadinejad*

Department of Mechanical Engineering, Yasouj University, Yasouj, Iran

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Abstract. This paper numerically and analytically investigates a non-linear static, two-dimensional thermoelastic analysis in the radial and tangential directions of a cylindrical shell made of functionally graded materials. The dependence of material properties on temperature makes the heat governing equations non-linear. To obtain the temperature field analytically, the heat conduction equation is linearized and exactly solved using a linearizing transformation, then this exact solution is substituted in the Lamme-Navier equations, and the elasticity equations are numerically solved using a second-order central finite difference method, and displacement and stress distributions are obtained. Finally, the temperature field, stress, and displacement distributions are presented, and the effect of inhomogeneous parameters on them is examined and discussed. The correctness and accuracy of the exact analytical solution of the temperature field are illustrated by a comparison with the numerical solution. The results show a good agreement.

AMS subject classifications: 35Q79, 65E05, 76D05, 97N40

Key words: Non-linear heat conduction, temperature-dependent heat conductivity, functionally graded material, exact solution, hollow cylinder.

1 Introduction

Functionally graded materials (FGMs) are an advanced type of materials whose properties (mechanical, thermal, etc.) vary gradually and smoothly concerning their dimensions in space. These materials can be developed for certain industrial applications and purposes. Since most advanced engineering structures are needed to work under elevated mechanical loads and high temperatures. To meet this requirement, FGMs are one of the

*Corresponding author.
Emails: hameduvwt@gmail.com (H. Panahi-Kalus), moosaie@yu.ac.ir (A. Moosaie), mehrdadahmadinejad607@gmail.com (M. Ahmadinejad)

capable approaches to be used in such circumstances. On the other hand, most materials are prone to fail and creep under both mechanical and thermal loadings, i.e., steel, aluminum and ceramics, these mentioned materials withstand alone one kind of loads (mechanical or thermal). However, in most engineering applications such as aerospace in which the vehicle structure is subjected to both mechanical and thermal loadings, it is of major beneficial that materials be able to tolerate both types of loads simultaneously.

The FGMs are well-known to resist the mixed effect of mechanical and thermal loads. Additionally, the significance of FGMs is their advantageous application in high temperature and hazardous environments. According to different benchmarks, FGMs could be combined with a wide range of constitutions such as metal with ceramic, ceramic with ceramic, and ceramic with plastic. In engineering applications, for the sake of reducing the mechanical and thermal stress concurrently, it is common to use material such as metal on the low- temperature area and material like ceramic on the high-temperature one. Subject to the above applications the thermoelastic analysis of such materials is of great importance. So, in the following paragraphs, literature on the thermoelastic analysis of FGM structure is presented.

Liew et al. [1] provided an analysis of the temperature and thermal stresses in an FGM hollow circular cylinder using a novel limiting process solution. An accurate analysis approach has been proposed to tackle thick-walled spherical pressure vessels made of FGM systems subjected to internal pressure by You et al. [2]. For the first time, the non-linear oscillations and chaotic dynamics of the FGM rectangular plate subjected to the mixed transversal and in-plate excitations are studied by Hao et al. [3]. By considering material properties homogeneous and isotropic, Moosaie [4] analytically has investigated the non-Fourier heat conduction equation in a hollow sphere with time-dependent boundary conditions. Jabbari et al. [5, 6] have investigated the exact solution of one- and two-dimensional axisymmetric mechanical and thermal stresses for an FGM hollow cylindrical shells, graded by a power function, with the use of the Bessel functions. They considered temperature-independent properties and their proposed method can also be used for similar geometries such as a sphere. Shariyat et al. [7, 8] explored non-linear thermoelasticity, vibration, and stress wave propagation of thick-walled cylinders made of FGM assuming properties depend upon temperature. By assuming uncertain material properties subjected to mechanical shock load, Shahabian et al. [9] probed the analysis of the stochastic dynamics of an FG cylinder utilizing the Newmark finite difference approach with Monte Carlo simulation to solve the statistical response of the cylinder.

Dai et al. [10] have explored an exact solution of one-dimensional steady-state magnetothermoelastic stresses for an FGM hollow sphere. The dependency of properties on the radial component is considered as power functions of the sphere radius. An analytical method to study the non-linear static buckling for imperfect eccentrically stiffened FG cylindrical shells with temperature-dependent properties has been proposed by Duc et al. [11]. The generalized thermoelasticity in a thick-walled FGM cylinder with a relaxation time is studied in [12]. In [12] materials properties are considered a function of temperature and spatial in the radial coordinate. Due to the dependency on the temper-