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A Mesh Size Scaling Law with Reynolds Number for Large Eddy Simulation in Channel Flow

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Abstract. In this paper, a scaling law relating the mesh size to the Reynolds number was proposed to ensure consistent results for large eddy simulation (LES) as the Reynolds number was varied. The grid size scaling law was developed by analyzing the lengthscale of the turbulent motion by using DNS data from the literature. The wall-resolving LES was then applied to a plane channel flow to validate the scaling law. The scaling law was tested at different Reynolds numbers ($\mathrm{Re}_\tau = 395$, 590 and 1000), and showed good results compared to direct numerical simulation (DNS) in terms of mean flow and various turbulent statistics. The velocity spectra analysis shows the evidence of the Kolmogorov –5/3 inertial subrange and verifies that the current LES can resolve the bulk of the turbulent kinetic energy by satisfying the grid scaling law. Meanwhile, the near-wall turbulent flow structures can also be well captured. Reasonably accurate predictions can thus be obtained for flows at even higher Reynolds numbers with significantly lower computational costs compared to DNS by applying the mesh scaling law.

AMS subject classifications: 76F65

Key words: Large eddy simulation (LES), channel flow, turbulent flow, mesh size scaling law.

1 Introduction

Turbulence plays a dominant role in most engineering and natural flows. A turbulent flow is unsteady, chaotic and unpredictable, with the exact physical nature remaining mysterious. Thanks to the rapidly increasing computing power, Computational Fluid Dynamics (CFD) now offers a promising approach for calculating the relevant properties of turbulent flows. Direct numerical simulation (DNS) is one important method for the study of turbulent flows, which directly solves the Navier-Stokes equations (NS-equations) for all scales of motion. However, this approach is computationally too expensive, and is usually restricted to flows with relatively low Reynolds numbers. For

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numerical simulations with higher Reynolds numbers, turbulence is usually modelled. Reynolds averaged Navier–Stokes (RANS) models solve the Reynolds equations (mean-flow equations) to determine the mean velocity field by modelling the entire Reynolds stresses. RANS models have been used for various problems with turbulent flow involved, due to their ease of implementation and low computational cost. Large eddy simulation (LES) is another important approach, where the governing equations are solved for a filtered velocity field – representing the larger/grid scale flow motions. Meanwhile, the influence of the smaller/sub-grid scale (SGS) motions is represented by a model included in the filtered NS-equations. Thus, extensive calculations are avoided to explicitly solve the smaller-scale motions. Compared to RANS, LES has the advantage of describing the unsteady, grid-scale turbulent structures though it is computationally more expensive. Hence, it is a good choice to apply LES for solving the unsteady motion in turbulent flows to a certain extent of accuracy.

The earliest and simplest LES model was proposed by Smagorinsky [1], where the sub-grid scale (SGS) turbulent stress tensor is related to the resolved strain-rate tensor by a scalar eddy viscosity with a linear algebraic equation. Based on the linear eddy-viscosity assumption in the Smagorinsky model, more models are proposed by other researchers, including dynamic models [2–4], dynamic mixed models [5, 6], structure function models [7, 8], wall adapting local eddy viscosity model (WALE) [9], etc. More recently, the anisotropic minimum-dissipation (AMD) model has been proposed by Rozema et al. [10] and evaluated in OpenFOAM by Zahiri and Roohi [11, 12], which considers the effect of various directions in computing sub-grid stress and is capable of operating in transitional flows. In the above models, either constant model coefficient or dynamic model coefficient is adopted–an operation called test filtering is commonly used to evaluate the dynamic coefficient. Transport-equation model is another important LES approach, where transportation equations for the SGS terms are formulated, accounting for the historic and non-local effect of SGS kinetic energy due to production, dissipation and diffusion. Representative works on the transport-equation LES model are given by Deardorff [13], Schumann [14], Yoshizawa and Horiuti [15], Ghosal et al. [16], Fureby et al. [17], Krajnović and Davidson [18], Gallerano et al. [19]. The SGS turbulent stress tensor is explicitly modelled in the above introduced models, while Boris et al. [20] advocated to solve the filtered NS-equations without using an explicit SGS model and to use the inherent dissipation from the discretization scheme as an implicit SGS model. This approach is known as MILES (Monotone Integrated Large Eddy Simulation). Other representative LES calculations with this approach can be found in the studies by Tamura and Kuwahara [21], Knight et al. [22], Urbin and Knight [23]. The reader may refer to Meneveau and Katz [24], Pope [25], Yang [26] for more details about LES modelling, where the performance of various models is evaluated and discussed.

Although substantial efforts have been made to develop various SGS models in the last few decades, they all have limitations–a model which works very well for one type of problem may turn out to be unsuitable for another type of problem. In addition, the accurate simulation of near-wall flow regions is essential in many practical engineering