Advances in Applied Mathematics and Mechanics
Adv. Appl. Math. Mech., Vol. **14**, No. 6, pp. 1333-1356

DOI: 10.4208/aamm.OA-2021-0334
December 2022

Piecewise Spectral Collocation Method for Second Order Volterra Integro-Differential Equations with Nonvanishing Delay

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Received 6 November 2021; Accepted (in revised version) 7 March 2022

Abstract. In this paper, the piecewise spectral-collocation method is used to solve the second-order Volterra integral differential equation with nonvanishing delay. In this collocation method, the main discontinuity point of the solution of the equation is used to divide the partitions to overcome the disturbance of the numerical error convergence caused by the main discontinuity of the solution of the equation. Derivative approximation in the sense of integral is constructed in numerical format, and the convergence of the spectral collocation method in the sense of the L^∞ and L^2 norm is proved by the Dirichlet formula. At the same time, the error convergence also meets the effect of spectral accuracy convergence. The numerical experimental results are given at the end also verify the correctness of the theoretically proven results.

AMS subject classifications: 65M10, 78A48

Key words: Second-order Volterra type integro-differential equation, delay function, piecewise spectral-collocation method.

1 Introduction

Second-order Volterra integro-differential equations (VIDEs) have long appeared in mathematical models of physical phenomena and biological phenomena, which has led many scholars to develop a theoretical and numerical analysis of these equations. For some early research results, such as the general linear method [7], the linear multistep

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method [8, 9], the Runge-Kutta method [10, 12]. Over the years, polynomial spline collocation method [13–16, 27, 28], spectral Galerkin method [20] and Bologna [29] have developed an asymptotic solution for first-order and second-order VIDEs containing arbitrary kernels. In [24, 37], the collocation method is used to approximate second-order VIDEs. In the recent papers [42–44], an hp-version of the spectral method has been proposed and analyzed for VIDEs.

The spectral method has been used in applied mathematics and scientific calculations to numerically solve certain partial differential equations (PDEs) [31–34]. In practice, the spectral method has high-precision convergence, the so-called "exponential convergence".

Delay VIDEs have many practical applications, such as competitive ecosystem [1], biology and Species model [2–4], virus transmission [5], and so on. For more information on the application of VIDEs with delay in species models, please refer to reference [6].

Remove the integral term in the delayed VIDEs to get the delayed differential equation(DDEs). The DDEs model appears in many practical problems, such as tumor growth models [38], species dynamics systems [39], hepatitis virus infection models [40], and toxic species presence diffusion models [41]. Reference [39] contains a lot of literature on the application of DDEs.

The literatures using spectral methods for solving delay VIDEs are [17–19, 21, 22, 24–26, 35]. The numerical methods used in these articles are used to solve the vanishing delay VIDEs. So far, there are few numerical methods for solving the nonvanishing delay type VIDEs that can make the numerical error converge to the spectral accuracy. When the VIDEs with nonvanishing delay are numerically solved, it is necessary to overcome that the solution of the equation has a main discontinuity point, which is inconsistent with the spectral method's requirement for the global smoothness of the equation solution. In the literature [36], the author gives a method of slicing the spectrum for solving VIDEs with nonvanishing delay first-order VIDEs, and gives proof of convergence. So far, few scholars have studied the spectral approximation of second-order VIDEs for the piecewise spectral-collocation method.

The second-order Volterra integro-differential equation with nonvanishing delay considered here is as follows:

$$\begin{aligned} y''(t) = & a_1(t)y(t) + a_2(t)y'(t) + b_1(t)y(\theta(t)) + b_2(t)y'(\theta(t)) + g(t) \\ & + \int_0^t K_1(t,s)y(s)ds + \int_0^t K_2(t,s)y'(s)ds \\ & + \int_0^{\theta(t)} R_1(t,s)y(s)ds + \int_0^{\theta(t)} R_2(t,s)y'(s)ds, \end{aligned} \tag{1.1a}$$

$$y(t) = \phi(t), \quad y'(t) = \varphi(t), \quad t \in [\theta(0), 0]. \tag{1.1b}$$

The functions $y(t)$, $t \in (0,T]$ are unknown functions, and $y \in C^{m+1}([0,T])$. Assume func-