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A Conservative Upwind Approximation on Block-Centered Difference for Chemical Oil Recovery Displacement Problem

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Abstract. A kind of conservative upwind method is discussed for chemical oil recovery displacement in porous media. The mathematical model is formulated by a nonlinear convection-diffusion system dependent on the pressure, Darcy velocity, concentration and saturations. The flow equation is solved by a conservative block-centered method, and the pressure and Darcy velocity are obtained at the same time. The concentration and saturations are determined by convection-dominated diffusion equations, so an upwind approximation is adopted to eliminate numerical dispersion and nonphysical oscillation. Block-centered method is conservative locally. An upwind method with block-centered difference is used for computing the concentration. The saturations of different components are solved by the method of upwind fractional step difference, and the computational work is shortened significantly by dividing a three-dimensional problem into three successive one-dimensional problems and using the method of speedup. Using the variation discussion, energy estimates, the method of duality, and the theory of a priori estimates, we complete numerical analysis. Finally, numerical tests are given for showing the computational accuracy, efficiency and practicability of our approach.

AMS subject classifications: 65N12, 65N30, 65M12, 65M15

Key words: Chemical oil recovery, upwind block-centered difference, fractional step difference, elemental conservation, convergence analysis.

1 Introduction

Oil exploration plays an important rule in industrial engineering fields, while the underground crude oil becomes less. New challenges of exploration techniques appear in

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this project, such as how to decrease the cost and increase the recovery efficiency at the present oilfields. At present, a new recovery technique, chemical agents used during the displacement, is generalized. Under the influences of driving fluids with addition of chemical agents, crude oil is migrated and accumulated easily through the underground media. Some chemical additives usually include polymer, surface active agent and alkali. This is called "chemical oil recovery" [1–6]. The mathematical model is formulated by a nonlinear system of partial differential equations [7–11]. It is important to find efficient numerical methods for simulating how the underground fluids flow and oil is displaced more accurately. In this paper, the physical natures and the characters of mathematical model are considered carefully, then a kind of upwind method with block-centered difference and fractional step difference together is discussed. Numerical analysis and experimental tests are shown.

The mathematical model with initial-boundary conditions is given

$$-\nabla\cdot\Big(\frac{\kappa(X)}{\mu(c)}\nabla p\Big)\equiv\nabla\cdot\mathbf{u}=q(X,t)=q_I+q_p,\quad X=(x,y,z)^T\in\Omega,\quad t\in J=(0,T],\tag{1.1a}$$

$$\mathbf{u}=-\frac{\kappa(X)}{\mu(c)}\nabla p,\qquad X\in\Omega,\qquad t\in J,\tag{1.1b}$$

and

$$\phi\frac{\partial c}{\partial t}+\mathbf{u}\cdot\nabla c-\nabla\cdot(D(\mathbf{u})\nabla c)+q_Ic=q_Ic_I,\qquad X\in\Omega,\quad t\in J,\tag{1.2a}$$

$$\phi\frac{\partial}{\partial t}(cs_\alpha)+\nabla\cdot(s_\alpha\mathbf{u}-\phi c\kappa_\alpha\nabla s_\alpha)=Q_\alpha(X,t,c,s_\alpha),\quad X\in\Omega,\quad t\in J,\quad \alpha=1,2,\cdots,n_c,\tag{1.2b}$$

where Ω is a bounded domain in R^3. The pressure, Darcy velocity, the concentration of water and the saturations of different chemical components are denoted by $p(X,t)$, $\mathbf{u}=(u_1,u_2,u_3)^T$, $c(X,t)$ and s_α ($\alpha=1,\cdots,n_c$) respectively. Other major parameters are interpreted as follows

- $q(X,t)$, the quantity, usually defined by the production q_p and the injection q_I, i.e.,
$$q(X,t)=q_I(X,t)+q_p(X,t),$$
- c_I, the concentration of injected fluid,
- $\phi(X)$, the porosity of rock,
- $\kappa(X)$, the absolute permeability,
- $\mu(c)$, the viscosity of mixture dependent on c,
- $D=D(\mathbf{u})$, the diffusion coefficient, defined by molecular diffusion and mechanical dispersion
$$\mathbf{D}(X,\mathbf{u})=\phi d_m\mathbf{I}+|\mathbf{u}|^\beta\begin{pmatrix}d_l&0&0\\0&d_t&0\\0&0&d_t\end{pmatrix}=\begin{pmatrix}d_x(\mathbf{u})&0&0\\0&d_y(\mathbf{u})&0\\0&0&d_z(\mathbf{u})\end{pmatrix},\tag{1.3}$$