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A Comparative Study on Polynomial Expansion Method and Polynomial Method of Particular Solutions

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Abstract. In this study, the polynomial expansion method (PEM) and the polynomial method of particular solutions (PMPS) are applied to solve a class of linear elliptic partial differential equations (PDEs) in two dimensions with constant coefficients. In the solution procedure, the sought solution is approximated by the Pascal polynomials and their particular solutions for the PEM and PMPS, respectively. The multiple-scale technique is applied to improve the conditioning of the resulted linear equations and the accuracy of numerical results for both of the PEM and PMPS. Some mathematical statements are provided to demonstrate the equivalence of the PEM and PMPS bases as they are both bases of a certain polynomial vector space. Then, some numerical experiments were conducted to validate the implementation of the PEM and PMPS. Numerical results demonstrated that the PEM is more accurate and well-conditioned than the PMPS and the multiple-scale technique is essential in these polynomial methods.

AMS subject classifications: 35C11, 65N35

Key words: Pascal polynomial, polynomial expansion method, polynomial method of particular solutions, collocation method, multiple-scale technique.

1 Introduction

In the past years, there has been an increasing interest in the idea of meshless numerical methods for solving partial differential equations (PDEs) as the computational costs

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for constructing meshes can be minimized. Basically, meshless numerical methods can be classified into boundary and domain types. For examples, boundary-type meshless numerical methods include the method of fundamental solutions [1, 2], the modified Trefftz collocation method [3–5], the regularized meshless method [6, 8], the boundary knot method [9, 10], the boundary point interpolation method [11, 12], the wave based method [13, 14], etc. One great advantage of this type of meshless numerical methods is that the dimensionality of the problem is reduced by one.

On the other hand, domain-type methods contain the radial basis function collocation method [15–18], the smoothed particle hydrodynamics [19, 20], the diffuse element method [21, 22], the element-free Galerkin method [23, 24], the meshless local Petrov-Galerkin method [25, 26], the reproducing kernel particle method [27, 28], the polynomial expansion method (PEM) [29], the localized method of fundamental solutions (LMFS) [30], the localized boundary knot method (LBKM) [31] and others. Recent developments on the meshless numerical methods have been comprehensively reviewed by several articles [32–35].

When the PEM is applied for solving PDEs, the Pascal polynomials are utilized to approximate the sought solutions. However, it is rarely considered in the earlier literature as the resultant linear equations of the PEM are ill-conditioned and numerically unstable. In order to remedy these difficulties, Liu and Atluri [36] introduced a characteristic length into the PEM to implement the multiple-scale PEM. They demonstrated that the multiple-scale PEM can improve not only the conditioning of the resultant linear equations but also the accuracy of numerical results. Sequentially, Liu and Young [37] and Liu and Kuo [38] utilized the multiple-scale PEM to solve the Cauchy problems of Stokes flows and the linear elliptic PDEs in complex domains, respectively. Then, the multiple-scale PEM was successfully applied to solve nonlinear heat conduction problems and three-dimensional PDEs with variable coefficients by Chang [39] and Liu et al. [40], respectively. Recently, Oruç [41–43] applied the multiple-scale PEM for solving the Berger equation, convection-diffusion problems, and elliptic PDEs with nonlocal boundary conditions.

Alternatively, the method of particular solution (MPS) are usually used for obtaining the particular solutions of the considered PDEs after the homogenous solutions are solved by boundary-type numerical methods [44, 45]. It is known that the success of the MPS depends on the availability of the closed-form particular solution associated with the basis function and the considered differential operator. Golberg et al. [46] is among the first to derive the closed-form particular solutions of the Pascal polynomials associated with the harmonic and Helmholtz operators. Sequentially, Karageorghis and Irene [47] extended the prescribed particular solutions to Chebyshev polynomials and biharmonic operators. Then, Tsai and his coauthors [29,48] derived the closed-form particular solutions of the polynomials associated with the poly-harmonic and poly-Helmholtz operators.

Recently, Dangal et al. [49] proposed a new strategy by directly using the closed-form particular solutions of the Pascal polynomials to approximating the sought solutions of the considered PDEs. Sequentially, the prescribed polynomial method of particular so-