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Isotropic Cloak Materials Design Based on the Numerical Optimization Method of the Inverse Medium Problem

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Abstract. In recent years, various kinds of cloak devices were designed by transformation optics, but these cloak metamaterials are anisotropic and difficult to manufacture. In this paper, we designed the isotropic cloak metamaterials based on the numerical method of the optimization theory to the inverse medium problem. This method has universality, and it is not limited by the shape and type of the cloak device. This isotropic material is easier to manufacture in practice than anisotropic material. A large number of numerical results show the effectiveness of the method.

AMS subject classifications: 35R30, 65N30

Key words: Inverse medium problem, isotropic cloak materials, finite element simulations.

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1 Introduction

Metamaterials are man-made material that have special effects on wave due to their special microstructure. Artificial metamaterials are different from ordinary materials in nature. Therefore, wave propagation can be controlled by humans through specially designed material parameters. The propagation theory was derived from Veselago's hypothetical chiral materials [32]. In this hypothetical material, both permeability and permittivity are negative, the wave propagation presents negative refraction [33], and the electric field, magnetic field and wave vector in the dielectric present a left-handed relationship, so it is also called left-handed materials [34]. The constitutive parameters of artificial metamaterials are double negative, which has great application value [31]. Therefore, in recent years, there have been great advances in the structure and design of chiral materials [27–30].

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Invisibility is also a type of practical application developed by artificial metamaterials. Generally, there are two types of material designs: external invisibility and internal invisibility [42]. External invisibility means that the metamaterial does not cover the object and interacts with the experimental area outside, so that the material does not affect wave propagation [10]. The internal invisibility is to cover the experimental area with cloak area. Generally, a multi-layer metamaterial cloak is used to interact with experimental materials, thus the field does not change after passing through the area, which contributes to it that the experimental area does not affect wave propagation [2].

In 1988, the transmission eigenvalue problem in scattering theory was studied by Colton and Monk [25], and some wave propagation equations of common fields could be described by a group of transmission eigenvalue equations. For general wave propagation problems, one of the problems is the electromagnetic problem derived from Maxwell's equations. To make electromagnetic waves propagate invisibly, the transformation optics (TO) method was proposed by Pendry in negative refraction theory in 2006, which was usually applied to the invisible propagation of electromagnetic waves [4–6]. It can be derived from the TO theory that the actual material is a double negative electromagnetic metamaterial invisibility cloak composed of a Split-ring resonator [7, 38]. This kind of resonator can be designed in a carpet [8], circle or rectangle [9]. The wave propagation can also be acoustic waves. From the form invariance of the acoustic wave Helmholtz equation, the acoustic cloak can also be designed through a similar TO theory derivation process [1, 11, 12]. There are some methods based on the development of plasmonic materials, which are used to directly design invisible cloaks [52, 55]. These include external invisibility, placing a plasmon resonance object to counteract external fields [54]. These also include internal invisibility, such as the plasmonic invisible method based on scattering cancellation, which designs a plasmonic shell to compensate for scattering [53]. Different from the direct problem, the material is designed so that the coverage area of the cloak does not affect the field propagation [13]. There are other methods based on the inverse problem to derive the required material parameters by solving the equation, which still achieves the invisible cloaks. For example, researcher Liu [47–49] uses TO methods to reshape obstacles in acoustics and electromagnetic scattering, Bao and Kohn [15, 16] design an interesting acoustic invisible material based on the inverse problem theory, where the good idea is to add a sound-hard layer to the cloak.

The use of Fourier transform to transform the wave equation into a time-harmonic equation was described by Ammari [20, 21]. Converting general equations to time-harmonic equations can eliminate the time term, which greatly simplifies mathematical processing. Using continuous basis function, numerical experiments are done in the case of discarding the second-order error, and the use of finite elements to solve time-harmonic equations has also been studied by Nedelec [22, 23]. Except for the continuous case, the discontinuous Galerkin method can be used to solve the discrete case for the electromagnetic inverse problem [44]. In recent years, some good results are obtained in the inverse problem of the heat equation, electromagnetic and acoustic equation [17, 18, 36, 37]. The use of conjugate gradient(CG) method to solve the inverse prob-