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# Finite Element Method Coupling Penalty Method for Flexural Shell Model

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**Abstract.** In this paper, we propose a conforming finite element method coupling penalty method for the linearly elastic flexural shell to overcome computational difficulties. We start with discretizing the displacement variable, i.e., the two tangent components of the displacement are discretized by using conforming finite elements (linear element), and the normal component of the displacement is discretized by using conforming Hsieh-Clough-Tocher element (HCT element). Then, the existence, uniqueness, stability, convergence and a priori error estimate of the corresponding analyses are proven and analyzed. Finally, we present numerical experiments with a portion of the conical shell and a portion of the cylindrical shell to verify theoretical convergence results and demonstrate the effectiveness of the numerical scheme.

**AMS subject classifications**: 65N12, 65N15, 65N30

**Key words**: Flexural shell, conforming finite element method, penalty method, conical shell, cylindrical shell.

## 1 Introduction

Research on shell model, especially the special geometry shell (such as: conical shell, cylindrical shell, spherical shell, hyperbolic shell, ring plate) and its combined structure are widely used in aerospace [1], engineering buildings [2], biomedical materials [3], marine engineering [4], nuclear industry [5] and many other engineering fields. Therefore, the study of elastic shells is one of the most important branches of the theory of elastic models [6]. Owing to the complexity of its geometry, load and boundary conditions, the analytical solutions cannot be directly obtained for all kinds of shell theory, which makes the numerical method of the finite element method [7] useful in the analysis of actual shell structures.

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The study of the statics shell traces back to the late 1960s. Based on the Kirchhoff-Love hypothesis and the Reissner-Mindlin hypothesis, Koiter [8,9] and Naghdi [10] used two-dimensional linear elastic models to approximate three-dimensional linear elastic shell [11] problems, respectively. For the first time, Ciarlet et al. [12–14] gave mathematical proofs of the membrane shell, the flexural shell and the Koiter shell. In [15], Ciarlet gave an introduction to membrane shells, flexural shells (also, cf. [16]) and Koiter shells (also, cf. [17]): definitions, theories and examples, which further complete the theory of static shells. Ciarlet-Mardare-Shen [18] discussed the Donati compatibility condition of the flexural shell through internal research, that is, as a quadratic minimization problem with measurable linear variation, the change of the curvature tensor of the neutral plane of the shell as a new unknown. For the problem of numerical approximation of the static elastic shells [19], the finite element method is the main method for shell structure analysis and one of the most effective methods.

In terms of static numerical calculation, static model generally adopts methods of conforming finite element, non-conforming finite element and mixed finite element. Conforming finite element [20] refers to the requirement that the function in the finite element space satisfies the continuity when using the finite element method for numerical calculation. For example, when solving second-order differential equations, it is required that the piecewise function in finite element space is continuous in the region; when solving fourth-order differential equation, the function in finite element space and its first order partial derivatives are required to be continuous. Bernadou-Boisserie [21] used the conforming element to approximate the continuous finite element such as the Koiter linear problem [22]. Non-conforming finite element [23] means that there is no requirement for the continuity of the function, that is, the discrete second-order differential equation does not need to be continuous, and the first derivative of the fourth-order differential equation does not need to be continuous. Compared to the conforming finite element, it uses less degrees of freedom, especially for the fourth-order differential equation. Kikuchi [24] established the convergence of the continuous solution of the Kirchhoff-Love plate model D.K.T.. A mixed finite element can approximate two vector variables and one scalar variable at the same time. And it can have a higher approximation order, and it requires two different finite element spaces, and the inf-sup condition must be met to ensure its stability. Brezzi-Fortin [25] performed a mathematical analysis of the mixed finite element.

The main difficulty of the numerical scheme for the flexural model is that we should construct a discretization space with a constraint condition, which is hard to implement. In this paper, we use the conforming finite element method coupling penalty method to discretize the flexural shell model [26]. In order to perform numerical simulation, we test the existence, stability, uniqueness, convergence and a priori error estimation of the corresponding solutions for displacement variables discretized. Finally, the numerical experiments [27] are carried out on the conical shell [28] and the cylindrical shell under particular boundary conditions. The displacement of the conical shell and the cylindrical shell under different grids are calculated. The numerical experiments show the convergence and validity of the finite element discrete scheme.