Advances in Applied Mathematics and Mechanics
Adv. Appl. Math. Mech., Vol. **13**, No. 3, pp. 619-644

DOI: 10.4208/aamm.OA-2020-0143
June 2021

Lattice Boltzmann Study of the Steady-State Relative Permeabilities in Porous Media

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Received 19 May 2020; Accepted (in revised version) 24 June 2020

Abstract. A multiple-relaxation-time (MRT) lattice Boltzmann model (LBM) is used to study the relative permeabilities in porous media. In many simulations in the literature, usually the periodic boundary condition at inlet and outlet and a uniform pressure gradient were applied to measure the relative permeabilities. However, it is not consistent with the pressure or velocity boundary conditions in the real experiments and may lead to unphysical results. Here using the convective outflow and constant velocity boundary conditions at outlet and inlet, respectively, we can simulate the real experimental setup. Meanwhile, the distribution of the two phases at the outlet can be resolved. The effects of wettability, initial saturation, viscosity ratio ($M \in (1,50)$), capillary number ($Ca \in (10^{-4},10^{-2})$) and micro two-phase distribution at the inlet on permeabilities are investigated comprehensively. It is found that generally speaking, the strong wetting, drainage, larger Ca, and larger M may result in a larger relative permeability of the non-wetting phase. Different flow pattern, the lubrication effect of the wetting phase that attaches to the wall, and influence of stagnant pores may contribute to the feature. The study is helpful to further develop the LBM to simulate the real experimental process.

AMS subject classifications: 76S05, 76T99, 76M28

Key words: Lattice Boltzmann, multiphase, convective outflow, porous media, relative permeabilities.

1 Introduction

Numerical study of multiphase flow in porous media is of great benefit to engineering applications [1–3]. Numerous macroscopic numerical methods have been developed for solving the two-phase Navier-Stokes (N-S) equation [4], such as the front-tracking method, volume-of-fluid (VOF) method, level set method, and so on. The former three

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methods are the most popular ones. However, the front-tracking method is usually difficult to simulate interface coalescence or break-up [4, 5]. In the VOF and level set methods, usually the interface reconstruction step or the interface reinitialization is required, which may be non-physical or complex to implement [5]. Besides, numerical instability may appear when the VOF and level set methods are applied to simulate surface-tension-dominated flows in complex geometries [4].

In the last twenty years, the Lattice Boltzmann method (LBM) has been developed into a useful tool to solve two-phase flow in porous media [6–11]. The LBM is a mesoscopic method and easy to handle complex wall geometries. It is also an explicit method, which makes the code easy to parallelize. In the LBM, solving the Poisson equation is not required. Hence, it is more efficient than common macroscopic schemes.

There are many multiphase LBMs available in the literature [7]. However, quantitative numerical study shows that the Rothman and Keller (R-K) model is more accurate than the other models [12]. The model was firstly proposed by Rothman and Keller [13] and further developed by Gunstensen et al. [14] through introducing an extra binary fluid collision into the Lattice Boltzmann equation [14]. Latva-Kokko and Rothman [15] improved the recoloring step in the R-K model, which is able to reduce the lattice pinning effect and decrease the spurious currents [5, 16]. Now the recoloring step is widely used in applications of the R-K model [5, 12, 17]. Recently, Reis and Phillips developed a two-dimensional nine-velocity R-K model [18]. In the model, a revised binary fluid collision is proposed and is shown to be able to recover the additional term which accounts for surface tension in the N-S equation [18].

For the R-K model, the wetting condition, i.e., contact angle in the pore scale can be specified through setting the densities of the two fluids in the solid nodes. That is more simpler than its counterpart, the free-energy LB model, in which the gradient of the density near the wall has to be imposed. Due to this simplicity, the model has been applied to simulations of multiphase flows in porous media [8, 19]. Reducing spurious currents is another important issue for multiphase models. Here in the R-K method, the multiple-relaxation-time (MRT) collision model [20] is adopted. The present method is able to reduce spurious currents and improve numerical stability significantly. Those are good features for reproducing the capillary fingering phenomena for two-phase flow in porous media [21].

There are many numerical simulations for testing the relative permeability, e.g., [11, 22, 23]. That is very different from the measurement scheme in experiments. Due to periodic boundary condition, the flow at the outlet is supposed to flow back to the inlet boundary. In this setup, the porous media is supposed to be infinitely long in the flow direction. That is different from the reality. Besides, the relative permeability has been reported to be slightly negative in the preliminary results of [11,23,24], which is not physically valid. These values are likely a result of applying periodic boundary conditions to a nonperiodic porous medium [23].

Hence, non-periodic boundary conditions may be necessary to develop. In [25], two-phase immiscible displacements driven by constant pressure difference were simulated.