Advances in Applied Mathematics and Mechanics
Adv. Appl. Math. Mech., Vol. **13**, No. 1, pp. 191-202

DOI: 10.4208/aamm.OA-2020-0165
February 2021

FETD Study of the Wave Propagation in Chiral Metamaterials

Lei Yang[1,*], Fuhao Liu[2] and Wei Yang[3]

[1] *Faculty of Information Technology, Macau University of Science and Technology, Macau, China*
[2] *School of Mathematics and Computational Science, Xiangtan University, Xiangtan, Hunan 411105, China*
[3] *Hunan Key Laboratory for Computation and Simulation in Science and Engineering and School of Mathematics and Computational Science, Xiangtan University, Xiangtan, Hunan 411105, China*

Received 30 May 2020; Accepted (in revised version) 27 July 2020

Abstract. In this paper, we propose a finite element time-domain (FETD) method for the Maxwell's equations in chiral metamaterials (CMMs). The time-domain model equations are constructed by the auxiliary differential equations (ADEs) method. The source excitation method entitled total-field and scattered-field (TF/SF) decomposition technique is applied to FETD method for the first time in simulating the propagation of electromagnetic wave in CMMs, based on which a unified ADE-FETD-UPML-TF/SF scheme is proposed to simulate the wave in CMMs. The following properties of CMMs can be observed successfully from the numerical experiments based on our method, i.e., the ability of the polarization rotation, and the negative phase velocity. The amplitude of reflected wave can effectively be controlled by the physical parameters of CMMs.

AMS subject classifications: 78M10, 35Q61

Key words: Maxwell's equations, finite element time-domain (FETD), chiral metamaterials (CMMs).

1 Introduction

As one kind of artificial electromagnetic material, metamaterials are different from the ordinary materials by their special features. One of the most representative features of the metamaterials is their negative refractive index [9, 12], which can be obtained when the permittivity and permeability are simultaneously negative [8]. Because of their special

*Corresponding author.
Emails: leiyang@must.edu.mo (L. Yang), liufuhaoedu@163.com (F. H. Liu), yangweixtu@126.com (W. Yang)

electromagnetic properties, metamaterials have been widely concerned by the scientific community in recent years, and have been successfully applied to the design of radar, antenna, and other microwave devices.

Chirality is a geometric concept, which describes the property of an object or a system that it can be distinguished from its mirror image. There are many examples for the natural chiral materials such as shells, snails, DNA, and for the artificial chiral materials such as spring, spiral ladder. The chiral media [1] are the special materials which can produce the polarization rotation phenomenons. Among those chiral materials, the chiral metamaterial is a new kind of metamaterials with excellent optical activity and circular dichroism, which potentially provides a simpler approach to achieve the negative refraction [2]. The study on CMMs belongs to the category of subwavelength electromagnetics. The chirality can be obviously observed from the electromagnetic wave propagating in the chiral metamaterial. Many amazing electromagnetic characteristics can be generated by the interaction between electromagnetic wave and subwavelength structure, such as negative refractive index, sub-wavelength optical waveguides, circular dichroism, cloaking devices, polarization rotation, etc. [4–7].

Numerical simulation play an important role in the design and application of CMMs. There have been many numerical methods in the market for the purpose, such as the popular finite difference time domain (FDTD) method [1–3, 13], method of moments [15] and Transmission Line Matrix (TLM) algorithm [10, 14]. As one of the most successful numerical methods, the finite element method has been studied systematically in solving the temporal equations, and has been widely applied in a variety of areas such as computational fluid dynamics, computational electromagnetics and micromagnetics etc.. One outstanding feature is its ability on handling the complex geometry from the practical problems, which also is an attractive feature for the interface problems in the elctromagnetics. In [11, 16, 17], the finite element time domain (FETD) method has been proposed and explored in the study of electromagnetic (EM) waves.

Although the finite element method has many advantages, as far as we know, there is no work about FETD method which is used to simulate the propagation of electromagnetic wave in CMMs. In this paper, we study the FETD method of the Maxwell's equations for CMMs. Because CMMs belongs to the dispersive media, there have been many methods to deal with the time-domain problems of electromagnetic wave propagation in dispersive media, including auxiliary differential equations (ADEs) method [18], recursive convolution method, shift operator method, and Z-transform method. It is noted that the computational resource required by ADEs method is lower among those methods, so in this paper we use ADE-FETD method to solve the EM problems with both frequency dispersion and chirality at the same time. In general, when the wave source is outside the electromagnetic material, we need to design the incident wave with some special methods. Total-field and scattered-field (TF/SF) decomposition technique has attracted wide attention because of its superior performance on controling the incident error. It is worth to mentioning that TF/SF method has been successfully applied to electromagnetic field simulation in chiral media and metamaterials for FDTD method [2, 13].