Advances in Applied Mathematics and Mechanics
Adv. Appl. Math. Mech., Vol. **13**, No. 1, pp. 83-100

DOI: 10.4208/aamm.OA-2019-0354
February 2021

A Novel Immersed Boundary Method Implemented by Imposing Reconstructed Velocity on Virtual Boundary

Jingyang Wang and Chunhua Zhou*

Department of Aerodynamics, Nanjing University of Aeronautics and Astronautics, Nanjing, Jiangsu 210016, China

Received 11 December 2019; Accepted (in revised version) 11 May 2020

Abstract. In this paper, a novel immersed boundary method is presented for simulating incompressible flows governed by Navier-Stokes equations. A virtual boundary is formed by the cell edges (for two-dimensional cases) in the vicinity of the immersed boundary. In the domain with the virtual boundary, the governing equations can be solved in the conventional way. Reconstructed velocity is imposed on the virtual boundary, which is determined via the interpolation along the direction normal to the wall and in conjunction with the no-slip condition for the actual boundary. For "freshly cleared nodes" on the virtual boundary encountered in moving-boundary problems, pressure at the previous time step is reconstructed by solving the local simplified momentum equation. In the test case for an analytical solution, the local accuracy of pressure is verified to be of the second order. In order to further validate the present method, various flows over the stationary and/or moving circular cylinder and NACA0012 airfoil have been simulated. The obtained results agree well with the available numerical or experimental data in the published literatures.

AMS subject classifications: 76D05, 76M25

Key words: Immersed boundary method, boundary condition, convergence rate, solution reconstruction.

1 Introduction

The immersed boundary (IB) methods have attracted a lot of attention in the numerical simulation of moving-boundary flows. Using the IB methods, the governing equations for the flow of fluid can be solved on a fixed mesh and there is no need to update the mesh at each time step. Hence, the computational cost for mesh updating and solution transferring between the old mesh and the new one can be eliminated, and the errors due to solution transferring can also be avoided.

*Corresponding author.
Email: chzhou@nuaa.edu.cn (C. H. Zhou)

The IB method was firstly proposed by Peskin for simulating cardiac mechanics and associated blood flow [1]. The idea of his method is to transmit the effect of the solid boundary into the momentum equations as forcing terms, thereby the no-slip boundary condition is satisfied. Mittal and Iaccarino classified roughly the IB methods into two categories [2]: one is termed continuous forcing approach (diffuse interface approach) in which the forcing term is incorporated explicitly into the momentum equations before discretization and another is the discrete forcing approach (sharp interface approach) in which the forcing term is introduced explicitly or implicitly after discretization. The continuous forcing approach is very attractive for elastic solid boundaries while it inherently cannot provide a sharp representation of rigid boundaries.

The essence of many discrete forcing IB methods [3–5] is to reconstruct the solution at ”forcing points” in the vicinity of the immersed boundary via the local approximate form of solution so that the spatial discrete form of the governing equations can be closured and the no-slip boundary condition can be enforced. In the ghost-cell IB method [3] and the local domain-free discretization method [5], some exterior cell-centers or exterior nodes near the immersed boundary are selected to be forcing points. In the hybrid Cartesian immersed boundary (HCIB) method [4, 6–8], some interior cell-centers or interior nodes near the immersed boundary are selected. When the second order spatial approximation is employed for the governing equations, the global and local accuracy of velocity is of the second order and the global accuracy of pressure is also of the second order [9,10]. However, in the work of [10,11], the local accuracy of pressure is reported to be around the first order. In many other published literatures, e.g., [3, 8, 12], the authors do not provide the result of the local accuracy of pressure.

In this work, a novel immersed boundary method is proposed, in which the corrected boundary conditions are enforced on a virtual boundary and then the governing equations can be solved in the same way as the conventional boundary-conforming methods. The virtual boundary consists of the cell-edges of background mesh and the correction of boundary condition is implemented by reconstructing the velocity at the nodes of virtual boundary. Our numerical experiment will show that the local accuracy of pressure obtained by the present method can reach the second order.

The remainder of this paper is arranged as follows. In Section 2, the governing equations and the basic numerical schemes are presented. Section 3 is the main part of the paper, in which the treatment of the immersed boundary is described. In Section 4, a numerical experiment for the verification of the convergence rate is carried out and flows past a circular cylinder and a NACA0012 airfoil are simulated to further validate the present IB method. Finally, in Section 5, summary and conclusion are given.

2 Governing equations and basic numerical schemes

In this work, two-dimensional incompressible flows of the fluid of constant density are governed by Navier-Stokes equations in the following form