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Modeling of Ice-Water Phase Change in Horizontal Annulus Using Modified Enthalpy Method

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Abstract. Phase change in ice-water systems in the geometry of horizontal cylindrical annulus with constant inner wall temperature and adiabatic outer wall is modeled with an enthalpy-based mixture model. Solidification and melting phenomena under different temperature conditions are analyzed through a sequence of numerical calculations. In the case of freezing of water, the importance of convection and conduction as well as the influence of cold pipe temperature on time for the complete solidification are examined. As for the case of melting of ice, the influence of the inner pipe wall temperature on the shape of the ice-water interface, the flow and temperature fields in the liquid, the heat transfer coefficients and the rate of melting are analyzed. The results of numerical calculations point to good qualitative agreement with the available experimental and other numerical results.

AMS subject classifications: 80A22

Key words: Phase change, solidification, melting, enthalpy model, fixed grid, annulus.

1 Introduction

Modeling of solid-liquid phase change phenomena has received a lot of attention since it is a common occurrence in metallurgical processes, latent heat thermal energy storages, oceanography, food processing, nuclear reactor safety etc. The nature of solid-liquid phase change can take different forms [1–3]. In the case of water and pure substances, the phase-change transition occurs at a constant temperature and a smooth continuous front separates the solid and the liquid phase distinctively, whereby different physical properties characterize the phases. The solidification of metal alloys and other multicomponent systems occurs at a finite temperature interval, and therefore a phase-change region (the "mushy" zone) forms, containing the solid and the liquid phase at the equilibrium temperature. The solid phase in the two-phase region

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can be formed as a rigid dendrite structure with a complex shape, or in the form of free floating particles which can be advected with the flow through or out of the mushy region. During the solidification of waxes, polymers and glasses, the liquid and the solid phase are dispersed throughout the phase-change region without creating a distinct solid-liquid interface.

The position of the solid-liquid front which propagates as the phase change proceeds, cannot be determined in advance, and it has to be determined as an unknown variable, together with the temperature field in both phases and the fluid flow in the liquid phase and the mushy region. The temperature fields in the solid and the liquid phase can be obtained from the energy conservation equations for solid and liquid phase respectively, while the fluid flow in the liquid is usually calculated from the momentum and the mass balance equation in the liquid. The transition of the solid-liquid interface is determined based on the interface energy balance equation which includes latent heat (L) as the key parameter in the phase-change process. The fluid flow is usually driven by buoyancy forces caused by the thermal gradients in the liquid phase. Several alternative numerical methods have been proposed for the solution of heat transfer with phase change that is usually referred to as the Stefan problem [4]. A survey of various numerical techniques can be found in Crank [5] and Voller [6]. One of the key challenges that have been the focus of research in the past is tracking the transient phase change interface which is usually treated with moving or fixed grid methods.

The moving grid methods (or variable grid, deforming grid, front tracking methods) [7,8] assume the fixing of the solid-liquid interface in each time step and solving the conservation equations separately for the solid and the liquid phase. The interface position has to be calculated for each time step using the classical Stefan formulation. As the solid-liquid interface propagates with time, the separate numerical grids for the solid and the liquid phase need to be rearranged (deformed) in each time step, to ensure that the node points are always on the phase-change front, which requires special techniques for meshing the domain.

The fixed grid methods (or enthalpy methods) [9,10] for phase change problems are based on one set of equations for both phases. A review of fixed grid techniques for phase-change problems can be found in Voller [9]. The phase change is taken into account with the appropriate source terms in the momentum and energy equations. The fluid flow in the two-phase region is usually described by Darcy law [10].The phase-change material is treated as a mixture, with unique physical properties depending only on liquid fraction (g_s) as a parameter. The main advantage of these methods is that there is no need to explicitly track the solid-liquid interface.

In this paper, an enthalpy-based model is applied to simulate the solid-liquid phase change in water, for the geometry of cylindrical horizontal annulus. This kind of geometry is widely used for thermal energy storage applications. Thermal energy storage based on solid-liquid phase change is a well-known and widely used solution used in air conditioning facilities, known to be a cost-effective measure. A valuable reference resource on thermal energy storage systems and applications is a book by