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Local Collocation Approach for Solving Turbulent Combined Forced and Natural Convection Problems

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Abstract. An application of the meshless Local Radial Basis Function Collocation Method (LRBFCM) [22, 30–33] in solution of incompressible turbulent combined forced and natural convection is for the first time explored in the present paper. The turbulent flow equations are described by the low-Re number $k-\varepsilon$ model with Launder and Sharma [23] and Abe et al. [1] closure coefficients. The involved temperature, velocity, pressure, turbulent kinetic energy and dissipation fields are represented on overlapping 5-noded sub-domains through the collocation by using multiquadrics Radial Basis Functions (RBF). The involved first and second order partial derivatives of the fields are calculated from the respective derivatives of the RBF's. The involved equations are solved through the explicit time stepping. The pressure-velocity coupling is based on Chorin's fractional step method [11]. The adaptive upwinding technique, proposed by Lin and Atluri [27], is used because of the convection dominated situation. The solution procedure is represented for a 2D upward channel flow with differentially heated walls. The results have been assessed by achieving a reasonable agreement with the direct numerical simulation of Kasagi and Nishimura [20] for Reynolds number 4494, based on the channel width, and Grashof number 9.6×10^5. The advantages of the represented mesh-free approach are its simplicity, accuracy, similar coding in 2D and 3D, and straightforward applicability in non-uniform node arrangements.

AMS subject classifications: 76F60, 76M25, 76R05, 76R10, 65D05, 65M22 and 65M70
Key words: Turbulent combined convection, two-equation turbulence model, radial basis function, collocation, meshless method, upward channel flow.

1 Introduction

Meshless methods represent a particular class of numerical methods for solving engin-

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eering and science problems. They differ from the classical numerical methods such as the Finite Difference Method (FDM), the Finite Element Method (FEM), and the Boundary Element Method (BEM) in the principal characteristic that the solution is represented on a set of nodes which are not confined to coordinate lines (as in FDM), and no polygonisation of the domain (as in FEM) or boundary (as in BEM) is required. There is a strong development in this class of novel numerical methods, demonstrated by the emerging books [3, 4, 8, 12, 26, 28, 29] and conference proceedings [5, 13]. There exists a simple class of meshless methods, structured on collocation of the continuum fields by the radial basis functions [9]. The method has been pioneered by Kansa [18, 19] and since then experiences a very fast development [31]. The main disadvantage of the original Kansa's formulation of the method was its inability to cope with large-scale problems due to the involved ill-conditioned full collocation matrices. This drawback was overcome in an elegant way through the local version of the method-LRBFCM, where the collocation is made point-wise on a subsets of the nodes [31] instead on their entire set.

In the last century, a lot of research has been devoted towards understanding of the turbulent flows. In spite of those attempts, a general physical theory still does not exist. Numerically, those flows could be very well predicted by the direct numerical simulation (DNS) of the Navier-Stokes equations. Unfortunately, in the DNS very fine spatial discretization has to be used in order to model and track all eddies of the flow, especially the smallest ones. The applicability of the DNS is currently limited to very simple geometries and for turbulent flows with moderate Reynolds (Re) numbers [24]. Other turbulent models are mainly derived through the time-averaging of the Navier-Stokes (N-S) equations. Due to the nonlinearity of the time-averaged N-S equations, a closure problem arises (more unknowns than equations), which puts these family of models into the category of semi-empirical ones. Various models were proposed [36], which are rather old, but still in use nowadays. Probably the most known and representative is the family of two-equation $k-\varepsilon$ models, which are further divided into two groups, standard (ST) and low-Re (LRN) models. The ST $k-\varepsilon$ models use the wall-functions, while the LRN models use special closure coefficients to correctly predict the turbulent boundary layers. Better predictions are obtained with the LRN models, but a very fine disretization near the walls is required. In this work, the LRN $k-\varepsilon$ model is used with the closure coefficients proposed by Launder and Sharma (LS) [23] and Abe et al. (AKN) [1].

The experimental and numerical investigation of turbulent flow due to the natural convection still remains very challenging. A lot of efforts were put into solving the turbulent natural convection in a closed square cavity [2, 16, 17, 40], where the system is closed and the natural convection is the only mechanism which drives the turbulent flow. However, many applications in nature and industry characterise the open layout in which the combined turbulent forced and natural convection take place. These problems became more interesting for researchers after the first DNS data were available in the mid-nineties. Kasagi and Nishimura [20] performed DNS calculations of fully developed turbulent flow between two vertical parallel plates kept