Advances in Applied Mathematics and Mechanics
Adv. Appl. Math. Mech., Vol. **3**, No. 2, pp. 141-164

DOI: 10.4208/aamm.10-m1025
April 2011

# Efficient Numerical Valuation of Continuous Installment Options

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Received 21 April 2010; Accepted (in revised version) 1 July 2010
Available online 5 January 2011

**Abstract.** In this work we investigate the novel Kryzhnyi method for the numerical inverse Laplace transformation and apply it to the pricing problem of continuous installment options. We compare the results with the one obtained using other classical methods for the inverse Laplace transformation, like the Euler summation method or the Gaver-Stehfest method.

**AMS subject classifications**: 65M10, 78A48

**Key words**: Installment options, Black-Scholes equation, numerical inverse Laplace transform, Gaver-Stehfest method, Euler summation method, optimal stopping problem, non-monotonic stopping boundary.

## 1 Introduction

Starting from the ancient age people tried to hedge their trading risks. We can find the predecessors of trading options looking at the history of ancient Rome, Phoenicia, Greece. With time passing options became more and more popular, drawing not only the hedgers, but the speculators also. In 1848, the new page of options' history was written as the *Chicago Board of Trade* (CBOT) was set up and the options started being traded officially. Developing rather slowly the option market then got into the boom in the end of 1960's-middle 1970's caused by the opening of the *Chicago Board Options Exchange* (CBOE) and the appearing of the well-known Black-Scholes model. This was the time when the modern history of the options started. Since that moment the

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interest in options was growing: the volumes of trading have increased, variety of new options types has appeared.

What are options today? Basically, options represent the right to buy (sell) the asset by a predetermined price at a certain date. The predetermined price in the option contract is known as the *exercise price* or *strike price*, the date of contract expiring is called the *exercise date* or *maturity*. At the exercise date the option holder has the right to acquire (sell) an underlying asset by the strike price or spot price regardless what is more favorable to him. The holder has to pay for this right, that is why the option has a certain price, the so-called *option premium*. Options where the holder buys the asset is known as *call option*. If the holder sells the asset the option is called *put option*. Talking about the time of exercising, options might be of European or American type. While *European options* can be exercised only at maturity, *American options* can be exercised at any moment up to the maturity.

This work is devoted to one of the quite recently emerged options, the so-called *installment option*. Installment option is a financial derivative where the small initial premium is paid up-front and the other part of the premium is divided into the installments to be paid during the lifetime of the contract up to maturity. At each *installment date* the investor has the right to decide if he continues to pay for the contract or he terminates paying, allowing the option to lapse.

Nowadays, installment options are rather widely traded in the financial markets. They possess significant advantage over other options. It is the possibility to stop paying the premiums before the option is expired. Thanks to this property the companies carrying out the policy of investments can reduce their losses. Moreover, taking in mind the nature of the installment options we can find a number of other contracts similar to them: some life insurance contracts and capital investment projects might be considered as installment options (see Dixit and Pindyck [13]). Thomassen and Van Wouwe [30] applied the installment option in pharmacy comparing the development of a new drug, evolving 6 stages, with a 6-variate installment option; MacRae [24] modeled the employee stock option as an installment option, etc.

As for the way of payments there exist 2 types of installment options: discrete and continuous. The *discrete installment options* are investigated concisely in the works of Karsenty and Sikorav [20], Davis et al. [11, 12], Ben-Ameur et al. [5] and Griebsch et al. [18].

The treatment of the *continuous installment options* is more complicated. There only exists a few related works. Ciurlia and Roko [7] studied the American case applying the *multipiece exponential function* (MEF) method to derive an integral form of the option's value. Their applied technique suffers from a serious drawback, since the MEF method generates a discontinuity in the optimal stopping and early exercise boundaries. Alobaidi [2] analyzed the European case using the Laplace transformation to solve the free boundary problem. However, the method used is rather specific and not suitable for a numerical computation.

In the current paper we focus on the valuation of the continuous installment options starting from the paper of Kimura [21], who successfully applied the Laplace