Advances in Applied Mathematics and Mechanics
Adv. Appl. Math. Mech., Vol. **3**, No. 1, pp. 87-108

DOI: 10.4208/aamm.09-m0977
February 2011

Transient Waves Due to Mechanical Loads in Elasto-Thermo-Diffusive Solids

J. N. Sharma[1,*], N. K. Sharma and K. K. Sharma

[1] *Department of Applied Sciences and Humanities, National Institute of Technology, Hamirpur-177 005, India*

Received 5 August 2009; Accepted (in revised version) 1 June 2010
Available online 15 October 2010

Abstract. This paper deals with the study of transient waves in a homogeneous isotropic, solid halfspace with a permeating substance in the context of the theory of generalized elasto-thermodiffusion. The halfspace is assumed to be disturbed due to mechanical loads acting on its boundary. The model comprising of basic governing differential equations and boundary conditions has been solved by employing Laplace transform technique. Noting that the second sound effects are short lived, the small time approximations of solution for various physical quantities have been obtained and the results are discussed on the possible wave fronts. In case of continuous and periodic loads acting at the boundary, the displacement is found to be continuous at each wave front while it is discontinuous in case of impulsive load. The temperature and concentration fields are found to be discontinuous at all the wave fronts. The displacement, temperature change and concentration deviation due to impulsive, continuous and periodic mechanical loads have also been evaluated in the physical domain at all times by employing numerical inversion technique of integral transform. The computer simulated numerical results have been presented graphically in respect of displacement, temperature change and concentration deviation for brass. A significant effect of mass diffusion has been observed on the behaviour of mechanical and thermal waves.

AMS subject classifications: 76R50, 44A10, 34A45, 35L05

Key words: Diffusion, Laplace transform, small time approximations, wave fronts.

1 Introduction

In integrated circuit fabrication, the diffusion is used to introduce dopants in controlled amount into semiconductor substrate. It is used to form base and emitter in

*Corresponding author.
Email: jns@nitham.ac.in (J. N. Sharma), nishanithmr@gmail.com (N. K. Sharma), kks@nitham.ac.in (K. K. Sharma)

bipolar transistors, integrated resistors and source/drain in metal oxide semiconductor (MOS) transistor. Thermal diffusion utilizes transfer of heat across a thin liquid or gas to accomplish the process of isotope separation. Mechanical load allows us to estimate the strength of materials because it indicates where the limit of strength properties lie and can be taken to improve the practical properties. The phenomena find application in geophysics as most part of earth is solid and at elevated temperature atomic diffusion occur in many chemical and physical processes. Keeping in view the wide use of materials under high temperature in microelectronic industry, nuclear reactors, etc. the theoretical study of elasto-thermodiffusive material is an important task in solid mechanics.

The theory of thermoelasticity deals with effect of mechanical and thermal disturbance in an elastic body. Duhamel [1] and Neumann [2] introduced the theory of uncoupled thermoelasticity which inherited two defects namely (i) the mechanical state of elastic body have no effect on temperature and (ii) the heat equation being parabolic that predicts infinite speed of wave propagation which again contradict the physical facts. Biot [3] developed the coupled theory of thermoelasticity to eliminate the paradox inherent in classical uncoupled theory that elastic changes have no effect on the temperature and the heat equation, however, is of diffusion type predicting infinite speed of propagation. To account for the finite speed of wave propagation Lord and Shulman [4] introduced the theory of generalized thermoelasticity with one relaxation time in classical Fourier law of heat conduction ensuring finite speed of wave propagation of heat and elastic waves. The governing equation of motion and constitutive relations remain the same as those for coupled and uncoupled theories. This fact was further supported by the theory given by Green and Lindsay [5], who developed a temperature rate dependent thermoelasticity with entropy inequality of Green and Laws [6] by including temperature rate among the constitutive variables, without violating the classical Fourier law when body under consideration has a center of symmetry and it allows heat wave to travel with finite speed.

Nowacki [7–10] developed the theory of thermoelastic diffusion by using coupled thermoelastic model. Dudziak and Kowalski [11], Olesiak and Pyryev [12] respectively discussed the theory of thermodiffusion and influence of cross effect arising due to coupling in the field of temperature, mass diffusion and strain in an elastic cylinder. Sherief et al. [13] derived the basic governing equations namely, equations of motion, heat conduction and mass diffusion for generalized elasto-thermodiffusive solid. Danilovskaya [14, 15] treated the problems of thermal shock on the surface of halfspace for the first time and obtained its analytical solution in dynamic uncoupled thermoelasticity. Sherief et al. [16] has studied problem of halfspace whose surface is rigidly fixed and subjected to effects of thermal shock in the context of generalized theory of thermoelasticity. Sherief et al. [17] has also studied halfspace problem in theory of generalized thermoelastic diffusion when surface of the halfspace is assumed traction free and subjected to time dependent thermal shock. Singh [18, 19] has investigated reflection of P and SV waves from free surface of elastic solid in context of generalized thermoelastic diffusion. Sharma [20] considered propagation of