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A CG-Type Method for Inverse Quadratic Eigenvalue Problems in Model Updating of Structural Dynamics

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Abstract. In this paper we first present a CG-type method for inverse eigenvalue problem of constructing real and symmetric matrices M, D and K for the quadratic pencil $Q(\lambda) = \lambda^2 M + \lambda D + K$, so that $Q(\lambda)$ has a prescribed subset of eigenvalues and eigenvectors. This method can determine the solvability of the inverse eigenvalue problem automatically. We then consider the least squares model for updating a quadratic pencil $Q(\lambda)$. More precisely, we update the model coefficient matrices M, C and K so that (i) the updated model reproduces the measured data, (ii) the symmetry of the original model is preserved, and (iii) the difference between the analytical triplet (M, D, K) and the updated triplet $(M_{\text{new}}, D_{\text{new}}, K_{\text{new}})$ is minimized. In this paper a computationally efficient method is provided for such model updating and numerical examples are given to illustrate the effectiveness of the proposed method.

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Key words: Inverse eigenvalue problem, structural dynamic model updating, quadratic pencil, iteration method.

1 Introduction

The times-invariant second order differential system

$$M\ddot{x} + D\dot{x} + Kx = f(t), \tag{1.1}$$

where $x \in \mathbb{R}^n$ and $M, C, K \in \mathbb{R}^{n\times n}$, arises frequently in a wide scope of important appli-

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cations, including applied mechanics, electrical oscillation, vibro-acoustics, fluid mechanics, signal processing, and finite element discretization of PDEs. It is well known that if $x(t) = ve^{\lambda t}$ represents a fundamental solution to (1.1), then the scalar λ and the vector v must solve the *quadratic eigenvalue problem* (QEP)

$$(\lambda^2 M + \lambda D + K)v = 0. \tag{1.2}$$

The scalars $\lambda \in \mathbb{C}$ and the nonzero vectors $v \in \mathbb{C}^n$ are called, respectively, eigenvalues and eigenvectors of quadratic matrix polynomial $Q(\lambda)$. Together, (λ, v) is called an eigenpair of $Q(\lambda)$. It is well known that the $Q(\lambda)$ has $2n$ finite eigenvalues over the complex field, provided the leading coefficient matrix M is nonsingular.

There are two aspects of the QEP, namely the direct problem and the inverse problem deserve attention. The direct problem analyzes and computes the spectral information, hence deducing the dynamical behavior of the system from a priori known physical parameters such as mass, elasticity, inductance and capacitance. The inverse problem determines or estimates the parameters of the system from its observed or expected eigen-information. Both problems are of significant importance in application. In this article, we consider a special inverse quadratic eigenvalue problem (IQEP) which is quite common in practice–construct the quadratic pencil with only a few eigenvalues and their corresponding eigenvectors. The IQEP that is of interest to us can be formulated as follows:

(IQEP) *(Inverse Quadratic Eigenvalue Problem)* Construct a nontrivial quadratic pencil

$$Q(\lambda) = \lambda^2 M + \lambda D + K,$$

so that its matrix coefficients (M, D, K) are of all symmetry structure and $Q(\lambda)$ has a specified set $\{(\lambda_i, \phi_i)\}_{i=1}^m$ as its eigenpairs.

Since we are only interested in real matrices, it is natural to expect that the prescribed eigenpairs are closed under complex conjugation. To facilitate the discussion, we shall described the partial eigeninformation via the pair $(\Lambda, \Phi) \in \mathbb{R}^{m\times m} \times \mathbb{R}^{n\times m}$ of matrices where

$$\begin{aligned}
\Lambda &= diag\left(\begin{bmatrix} \alpha_1 & \beta_1 \\ -\beta_1 & \alpha_1 \end{bmatrix}, \cdots, \begin{bmatrix} \alpha_l & \beta_l \\ -\beta_l & \alpha_l \end{bmatrix}, \lambda_{2l+1}, \cdots, \lambda_m \right) \in \mathbb{R}^{m\times m}, \\
\Phi &= \left[\phi_{1R}, \phi_{1I}, \cdots, \phi_{lR}, \phi_{lI}, \phi_{2l+1}, \cdots, \phi_m \right] \in \mathbb{R}^{n\times m}.
\end{aligned}$$

Here a 2×2 block $\begin{bmatrix} \alpha_j & \beta_j \\ -\beta_j & \alpha_j \end{bmatrix}$ and the corresponding columns $[\phi_{jR}, \phi_{jI}]$ in Φ represent of store the complex conjugate pairs of eigenvalues $\alpha_j \pm i\beta_j$ and the corresponding eigenvectors $\phi_{jR} \pm \phi_{jI}$. The IQEP therefore amounts to solving the algebraic equation

$$M\Phi\Lambda^2 + D\Phi\Lambda + K\Phi = 0, \tag{1.3}$$

for the matrices M, D and K subject to symmetry structure.