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Stability of Two Conservative, High-Order Fluid-Fluid Coupling Methods

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Abstract. This paper investigates two methods of coupling fluids across an interface, motivated by air-sea interaction in application codes. One method is for sequential configurations, where the air code module in invoked over some time interval prior to the sea module. The other method is for concurrent setups, in which the air and sea modules run in parallel. The focus is the temporal representation of air-sea fluxes. The methods we study conserve moments of the fluxes, with an arbitrary order of accuracy possible in time. Different step sizes are allowed for the two fluid codes. An *a posteriori* stability indicator is defined, which can be computed efficiently on-the-fly over each coupling interval. For a model of two coupled fluids with natural heat convection, using finite elements in space, we prove the sufficiency of our stability indicator. Under certain conditions, we also prove that stability can be enforced by iteration when the coupling interval is small enough. In particular, for solutions in a certain class, we show that the step size scaling is no worse than $\mathcal{O}(h)$ in three dimensions of space, where h is a mesh parameter. This is a sharper result than what has been shown previously for related algorithms with finite element methods. Computational examples illustrate the behavior of the algorithms under a wide variety of configurations.

AMS subject classifications: 65M12, 86-08

Key words: Air-sea, atmosphere-ocean, fluid-fluid, partitioned time stepping, conservative coupling.

1 Introduction

This paper concerns algorithms used to resolve interactions between the atmosphere and ocean for applications like climate research, hurricane modeling and regional weather forecasting. Since the physical properties of the air and sea systems are very different, separate code modules are used to simulate these with independent, internal numerical methods. Another code module, called a *flux coupler*, handles communication between

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the air and sea modules. The coupler receives state data from the fluid modules that is used to compute fluxes of conserved physical quantities, which are then transferred between the fluids in the form of boundary conditions. Important examples are the Model Coupling Toolkit (MCT) used to connect the Weather Research and Forecasting (WRF) and Regional Oceanic Modeling System (ROMS) models (e.g., [12, 19]) and the related CPL7 coupler used in the Community Earth Systems Model (CESM) [7].

There are challenges to couple the modules. Direct resolution of the boundary layers of the fluids is prohibitively expensive and the air model bottom need not match the sea model top. Thus, special bulk formulae have been derived to model the fluxes in terms of the state values at the air and sea boundaries. Examples we consider for the fluxes of momentum and heat are given later by (2.6b) and (2.7), which are motivated by the paper [14]. Also, the computational grids for the air and sea are different, so some conservative remapping of fluxes is needed in space. The afforementioned coupling software includes some conservative, spherical remapping functionality. Time stepping methods are a challenge since the internal time steps of the air and sea codes are different but the bulk flux formulae use simultaneous state values. This paper is focused on the latter issue of the calculation of fluxes for time stepping purposes.

The temporal representation of fluxes is complicated by a lengthy list of mathematical, scientific and computational considerations. We study two coupling methods as reference points to help advance our understanding regarding these issues. The precise algorithms are defined in Section 4. Due to the complexity of the problem, we shall first provide a high-level explanation of the algorithms in Section 1.1. We will then provide more background and fully explain our current goals in Section 1.2.

1.1 Two coupling algorithms

We study two methods to couple the air and sea modules that we refer to as the *sequential* and *concurrent* modes. In the sequential mode, the air code module is run on the same set of processors as the sea code. Since the atmospheric dynamics are thought of as driving the ocean surface conditions, we assume the air code is run first. For the concurrent mode, the air and sea modules run simultaneously on different sets of processors.

We explain the methods to advance the air and sea modules from a time t to time $t+\Delta t$, but they may advance internally using different numbers of substeps. Assume that the algorithms have been run on a previous interval $[t-\Delta t,t]$ already. The flux computations depend on least-squares data reconstructions in time, using polynomials of a chosen order $k\geq 0$, which can then be evaluated at any desired time. Enough data points (substeps) must be available to form the chosen reconstructions, which are defined rigorously in Section 4.2. To illustrate the algorithms, consider the case $k=1$, with 8 air substeps and 2 sea substeps. In the following figures, flux and state values are pictured abstractly as heights along the vertical axes, with time on the horizontal axes. An arrow head points to data that is functionally dependent on the corresponding data at the tail; double arrows (with no tail) denote a two-way (implicit) dependence.