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On the Numerical Solution of Logarithmic Boundary Integral Equations Arising in Laplace's Equations Based on the Meshless Local Discrete Collocation Method

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Abstract. In this article, we investigate the construction of a meshless local discrete collection method suitable for solving a class of boundary integral equations of the second kind with logarithmic singular kernels. These types of boundary integral equations can be deduced from boundary value problems of Laplace's equations with linear Robin boundary conditions. The numerical solution presented in the current paper is obtained by moving least squares (MLS) approach as a locally weighted least squares polynomial fitting. The logarithm-like singular integrals appeared in the method are computed via a particular nonuniform Gauss-Legendre quadrature rule. Since the offered scheme is based on the use of scattered points spread on the solution domain and does not require any background meshes, it can be identified as a meshless local discrete collocation (MLDC) method. We also obtain the error bound and the convergence rate of the presented method. The new technique is simple, efficient and flexible for most classes of boundary integral equations. The convergence accuracy of the new technique is examined over four integral equations on various domains and obtained results confirm the theoretical error estimates.

AMS subject classifications: 45A05, 41A25, 65D10, 45E99

Key words: Laplace's equation, boundary integral equation, logarithmic singular kernel, discrete collocation method, moving least squares (MLS) method, error analysis.

1 Introduction

The main purpose of this article is to propose a computational scheme for solving the following logarithmic boundary integral equation of the second kind:

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$$
\begin{aligned}
&-\pi u(\mathrm{x})+\int_{\partial D} u(\mathrm{y})\left(g(\mathrm{y})\ln|\mathrm{x}-\mathrm{y}|+\frac{\partial \ln|\mathrm{x}-\mathrm{y}|}{\partial n_{\mathrm{y}}}\right)ds_{\mathrm{y}} \\
&=\int_{\partial D} g(\mathrm{y})\ln|\mathrm{x}-\mathrm{y}|ds_{\mathrm{y}}, \qquad \mathrm{x}\in\partial D,
\end{aligned}
\tag{1.1}
$$

where the region D is open, bounded and simply connected in $\mathbb{R}^2$, n_{x} denotes the outward unit normal vector on ∂D, $p(\mathrm{x})$ and $g(\mathrm{x})$ are given functions on ∂D with $p(\mathrm{x})\geq 0$ but $p\not\equiv 0$ and the unknown function $u(\mathrm{x})$ must be determined [10,24]. These types of integral equations occur as a reformulation of the boundary value problem for two-dimensional Laplace's equation with linear Robin boundary conditions i.e.,

$$
\begin{cases}
\Delta u(\mathrm{x})=0, & \mathrm{x}\in D\subset\mathbb{R}^2, \\
\dfrac{\partial u(\mathrm{x})}{\partial n_{\mathrm{x}}}+p(\mathrm{x})u(\mathrm{x})=g(\mathrm{x}), & \mathrm{x}\in\partial D.
\end{cases}
\tag{1.2}
$$

It should be noted that boundary integral equations with logarithmic kernels are also appeared in connection with other types of partial differential equations (PDEs) occurred in several branches of sciences such as electromagnetism, solid and fluid mechanics, seismology, atomic scattering, wave diffraction, heat transfer, etc. [10,15,16,21].

The Galerkin and collocation methods [10,12] are high usage schemes to approximate the solutions of boundary integral equations. Authors of [11] have investigated the piecewise polynomial collocation method for solving boundary integral equations. Legendre wavelets [24], Spline wavelets [49], biorthogonal wavelets [30], trigonometric wavelet [28,32] and Daubechies interval wavelets [48,55] have been used to obtain the numerical solutions of boundary integral equations. An iterative quadrature method [54] has been presented for solving the boundary integral equation deduced as a reformulation of some boundary value problems for the two-dimensional Helmholtz equation. A spectral Galerkin method [45] has been applied to solve a boundary integral equation which arises from the two-dimensional Dirichlet problem. Fourier-Nystrom discretization scheme [31] has been developed for boundary integral equations associated with the Helmholtz equation. The methods proposed in these papers usually require some quadrature formulae to estimate the logarithmic singular integral appeared in these schemes. Gauss-type quadrature rules have been appropriated for weakly singular integrals in [24,33]. A useful research work conducted by authors of [39] has investigated a cell structure together with logarithmical Gaussian quadrature schemes for the numerical integration of boundary integrals.

The MLS scheme as a general case of Shepard's method has been introduced by Lancaster and Salkauskas [35]. The MLS consists of a local weighted least squares fitting, valid on a small neighborhood of a point and only based on the information provided by its closet points. The MLS approach is recognized as a meshless method because it is based on a set of scattered points and consequently does not need any domain elements. The meshless methods have significant important applications in different problems of