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Development and Comparative Studies of Three Non-free Parameter Lattice Boltzmann Models for Simulation of Compressible Flows

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Abstract. This paper at first shows the details of finite volume-based lattice Boltzmann method (FV-LBM) for simulation of compressible flows with shock waves. In the FV-LBM, the normal convective flux at the interface of a cell is evaluated by using one-dimensional compressible lattice Boltzmann model, while the tangential flux is calculated using the same way as used in the conventional Euler solvers. The paper then presents a platform to construct one-dimensional compressible lattice Boltzmann model for its use in FV-LBM. The platform is formed from the conservation forms of moments. Under the platform, both the equilibrium distribution functions and lattice velocities can be determined, and therefore, non-free parameter model can be developed. The paper particularly presents three typical non-free parameter models, D1Q3, D1Q4 and D1Q5. The performances of these three models for simulation of compressible flows are investigated by a brief analysis and their application to solve some one-dimensional and two-dimensional test problems. Numerical results showed that D1Q3 model costs the least computation time and D1Q4 and D1Q5 models have the wider application range of Mach number. From the results, it seems that D1Q4 model could be the best choice for the FV-LBM simulation of hypersonic flows.

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1 Introduction

In recent years, the lattice Boltzmann method (LBM) has attracted growing attentions

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as an alternative approach to simulate fluid flows [1–4]. Unlike conventional numerical methods, which are based on discretization of macroscopic conservation equations, the LBM directly solves the kinetic equations at mesoscopic level. The appealing merits of LBM are the simple algebraic operation, linear convective terms and easy parallelism. Thanks to such advantages, LBM has got a lot of achievements in various fields, such as in isothermal incompressible flows [5, 6], turbulent flows [7, 8], multiphase flows [9–11], etc.

However in the field of compressible flow simulation, the applications of LBM are still limited. Among limited works [12–16], the discrete velocity Boltzmann equation (DVBE) is usually solved. Kataoka and Tsutahara [12, 13] firstly proposed a method that utilizes the Crank-Nicolson scheme to solve DVBE to obtain the solution of flow field. Later, Qu et al. [14] presented a second-order TVD scheme to solve DVBE. They also proposed an alternative scheme [15] that discretizes DVBE by finite volume method (FVM). Besides, Li et al. [16] introduced the implicit-explicit (IMEX) Runge-Kutta scheme, a recently developed numerical technique for stiff problems, to solve DVBE. It must be pointed out that DVBE is a set of partial differential equations. The number of unknowns in DVBE (same as the number of lattice velocities) is much larger than that (number of conservative variables) in the macroscopic governing equations. In addition, the time step used for solving DVBE is usually very small due to severe stability condition. These lead to the solution of DVBE very inefficient. Another concern is that the multi-dimensional compressible lattice Boltzmann (LB) model is much more complicated than its one-dimensional counterpart. For example, the expressions of 2D equilibrium distribution functions shown in [14] are very complicated. This brings inconvenience for the application to solve multi-dimensional problems.

To avoid direct solution of DVBE, and in the meantime, to avoid application of multi-dimensional LB model, Ji et al. [17] proposed a finite volume-based lattice Boltzmann method (FV-LBM). In the FV-LBM, the LBM is used to construct flux solver at the interface, while the FVM is used to discretize the macroscopic governing equations. Hence, the advantages of two methods are well combined, i.e., efficient calculation of flux vectors and accurate simulation of all compressible features including shock, contact discontinuity and rarefaction wave by LBM and geometric flexibility by FVM. Apparently, the computational cost and virtual memory required by FV-LBM are far less than those of DVBE-based solvers [12–16] as the result of lesser variables involved. Moreover, the local time step and implicit residual smoothing scheme can also be applied to improve the computational efficiency. In the FV-LBM, only one-dimensional (1D) LB model is applied. Its one-dimensional application is quite straightforward. For the multi-dimensional case, the 1D LB model is applied along normal direction of the cell interface to evaluate the normal flux vectors. The flux vectors in the tangential direction are evaluated by using the same way as used in conventional Euler/Navier-Stokes (N-S) solvers. It should be indicated that although the idea of FV-LBM is given in [17], its details are not clearly shown. This paper will make up this scarcity and give details of FV-LBM.

So far, in the application of FV-LBM, the 1D LB models of Kataoka and Tsuta-