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A Three-Dimensional Gas-Kinetic BGK Scheme for Simulating Flows in Rotating Machinery

Di Zhou, Zhiliang Lu* and Tongqing Guo

Department of Aerodynamics, College of Aerospace Engineering, Nanjing University of Aeronautics and Astronautics, Qinhuai District, Nanjing 210016, Jiangsu, China

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Abstract. This paper focuses on the development and application of a three-dimensional gas-kinetic Bhatnagar-Gross-Krook (BGK) method for the viscous flows in rotating machinery. For such flows, a rotating frame of reference is usually used in formulating the Navier-Stokes (N-S) equations, and there are two major concerns in constructing the corresponding BGK model. One is the change of the convective velocities in the N-S equations, which can be reflected through modification of the gas streaming velocity. The other one is the necessity to account for the effect of the additional Coriolis and centrifugal forces. Here, a specifically-designed acceleration term is added into the modified Boltzmann equation so that the source effects can be naturally included into the gas evolution process and the resulted fluxes. Under the finite-volume framework, the constructed BGK model is locally solved at each cell interface and then the numerical fluxes can be evaluated. When employing the BGK scheme, it is sometimes found that the calculated spatial derivatives of the initial and equilibrium distribution functions are sensitive to the mesh quality especially in complex rotating flow applications, which may significantly influence flux evaluation. Therefore, an improved approach for computing these slopes is adopted, through which the modeling capability for viscous flows is enhanced. For validation, several numerical examples are presented. The computed results show that the present method can be well applied to a wide range of flows in rotating machinery with favorable accuracy.

AMS subject classifications: 76U05

Key words: Gas-kinetic scheme, BGK model, non-inertial reference frame, acceleration term, rotating machinery.

1 Introduction

The study of flows in rotating machinery has been an attractive and challenging issue. Examples of such flows cover a wide range of scientific and engineering applications [1],

*Corresponding author.
Email: luzl@nuaa.edu.cn (Z. L. Lu)

such as rotating Couette flow, rotating cavity flow, and turbomachinery flow. There are significant differences between rotating and non-rotating fluid applications. The first one is the presence of centrifugal and Coriolis forces in a rotating flow, which could be responsible for the possible differences in dynamics and aerodynamic performance. Secondly, the complex secondary flow pattern appearing in rotating machinery can be another distinction. It is caused by the combined effects of boundary layer formation, wake interactions and rotation. For example, many types of secondary flows occur in turbomachinery, including tip leakage, passage vortex, and corner vortex. Besides, in some cases the effect of flow rotation may lead to complex toroidal vortices and even flow instabilities. A notable example is the Taylor-Couette flow, where axisymmetric and non-axisymmetric instabilities appear when the characteristic angular velocity is increased above certain thresholds. These distinguishing features all increase the difficulty of accurate modeling of flows in rotating machinery and thus set a higher demand for numerical methods.

Currently, most computational fluid dynamics (CFD) computations are based on solving Navier-Stokes equations. Under the finite-volume framework, the inviscid fluxes are usually evaluated using upwind schemes, while the viscous fluxes are evaluated employing central differences. Typical upwind schemes include flux difference splitting (FDS)-type and flux vector splitting (FVS)-type schemes, which are both derived from the discretization of the Euler equations. However, practice has shown that the computational accuracy and robustness may vary depending on the selected numerical scheme, especially in boundary layer and shock structure calculations. Furthermore, numerical instabilities are sometimes encountered, such as the carbuncle phenomena and odd-even decoupling. These difficulties may be further enhanced in simulating rotating flows, probably causing significant impact to performance calculation and prediction of secondary flow pattern. Generally, we can attribute them to the excessive or insufficient numerical dissipation, which is implicitly provided in the scheme but does not meet the requirement in various flow regions. There are, however, more physical reasons to explain this. It is demonstrated in [2] that due to the difference between physical and numerical fluid, the Euler equations may not be an appropriate physical model to describe the time evolution of numerical fluid in all situations. Also because most upwind schemes use quasi-one-dimensional approaches, they may show quite different behaviors in multidimensional cases [3].

In the recent decade, there has been a growing interest in constructing schemes based on the Boltzmann equation since it has a more fundamental physical basis. The gas-kinetic BGK scheme (BGK), which was first proposed by K. H. Prendergast and K. Xu [4], is probably one of the most popular Boltzmann-type methods. Because of its many merits such as the multidimensionality, the delicate dissipative mechanism and the positive property, the BGK scheme can give accurate and robust N-S solutions. Particularly in the discontinuous regions, it has proven to be capable of capturing a crisp and stable shock structure [5]. With efforts of scholars, many useful BGK schemes have been developed for a wide variety of flows, as reported in [6–10]. As for rotating fluid applications concerned in this work, it also shows great potential in improving flow simulation.