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The Multi-Step Differential Transform Method and Its Application to Determine the Solutions of Non-Linear Oscillators

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Abstract. In this paper, a reliable algorithm based on an adaptation of the standard differential transform method is presented, which is the multi-step differential transform method (MSDTM). The solutions of non-linear oscillators were obtained by MSDTM. Figurative comparisons between the MSDTM and the classical fourth-order Runge-Kutta method (RK4) reveal that the proposed technique is a promising tool to solve non-linear oscillators.

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1 Introduction

Vibration problems and most of scientific problems in mechanics are naturally nonlinear. The equations modeling all these phenomena and problems are either ordinary or partial differential equations.Most of them do not have any analytical solution excepting a restricted set of these problems. Some are solved via the analytical perturbation method [1], whereas some of them are solved by numerical techniques. In many case studies, similarity transformations are used to reduce the governing differential equations into an ordinary nonlinear differential equation. In most cases, these equations do not have analytical solution. Therefore, these equations should be solved via spec-

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cial techniques. In last years, new methods were used by some researchers to solve these sorts of problem [2–5].

Integral transform methods like the Laplace and the Fourier transform methods are extensively used in engineering problems. By using these methods, differential equations are transformed into algebraic equations which are easier to cope with. In fact, integral transform methods are more complex and difficult when applying to nonlinear problems. A different dealing method to solve non-linear initial value problems is the MSDTM [6]. Our motivation is to concentrate on the applications of the multistep differential transform method (MSDTM).It should be mentioned that one of the main advantages of the MSDTM is its ability in providing us a continuous representation of the approximate solution, which allows better information of the solution over the time interval.

On the other side, the Runge-Kutta method (RK4) will provide solutions in discretized form, only at two ends of the time interval, thereby making it complicated in achieving a continuous representation. We purpose to contrast the effectiveness of MSDTM against the well-known fourth-order Runge-Kutta method.

Nonlinear oscillator models have been extensively encountered in many areas of physics and engineering. These models have meaningful importance in mechanical and structural dynamics for the wide understanding and accurate prediction of motion. Since many practical engineering components comprise of vibrating systems are modeled by using oscillator systems, these systems are important in physics and engineering [7, 8].

2 Differential transform method

The differential transform technique is one of the semi-numerical analytical methods for ordinary and partial differential equations that uses the form of polynomials as approximations of the exact solutions that are sufficiently differentiable. The basic definition and the fundamental theorems of the differential transform method (DTM) and its applicability for various kinds of differential equations are given in [9–13]. For convenience of the reader, we present a review of the DTM. The differential transform of the kth derivative of function $f(t)$ is defined as follows:

$$F(k)=\frac{1}{k!}\Big[\frac{d^kf(t)}{dt^k}\Big]_{t=t_0}, \tag{2.1}$$

where $f(t)$ is the original function and $F(k)$ is the transformed function. The differential inverse transform of $F(k)$ is defined as

$$F(t)=\sum_{k=0}^{\infty}F(k)(t-t_0)^k. \tag{2.2}$$

From Eqs. (2.1) and (2.2), we get

$$f(t)=\sum_{k=0}^{\infty}\frac{(t-t_0)^k}{k!}\frac{d^kf(t)}{dt^k}\Big|_{t=t_0}, \tag{2.3}$$