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Doubling Algorithm for Nonsymmetric Algebraic Riccati Equations Based on a Generalized Transformation

Bo Tang[1,2], Yunqing Huang[1,*] and Ning Dong[3]

[1] *School of Mathematics and Computational Science, Hunan Key Laboratory for Computation and Simulation in Science and Engineering, Key Laboratory of Intelligent Computing and Information Processing of Ministry of Education, Xiangtan University, Xiangtan 411105, Hunan, China*
[2] *School of Science, Hunan City University, Yiyang 413000, Hunan, China*
[3] *School of Science, Hunan University of Technology, Zhuzhou 412000, Hunan, China*

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Abstract. We consider computing the minimal nonnegative solution of the nonsymmetric algebraic Riccati equation with M-matrix. It is well known that such equations can be efficiently solved via the structure-preserving doubling algorithm (SDA) with the shift-and-shrink transformation or the generalized Cayley transformation. In this paper, we propose a more generalized transformation of which the shift-and-shrink transformation and the generalized Cayley transformation could be viewed as two special cases. Meanwhile, the doubling algorithm based on the proposed generalized transformation is presented and shown to be well-defined. Moreover, the convergence result and the comparison theorem on convergent rate are established. Preliminary numerical experiments show that the doubling algorithm with the generalized transformation is efficient to derive the minimal nonnegative solution of nonsymmetric algebraic Riccati equation with M-matrix.

AMS subject classifications: 65F50, 15A24

Key words: Shift-and-shrink transformation, generalized Cayley transformation, doubling algorithm, nonsymmetric algebraic Riccati equation.

1 Introduction

Consider solving the nonsymmetric algebraic Riccati equation

$$XCX-AX-XD+B=0 \tag{1.1}$$

*Corresponding author.
Emails: huangyq@xtu.edu.cn (Y. Q. Huang), apriltb@163.com (B. Tang)

and its dual form

$$YBY-DY-YA+C=0, \tag{1.2}$$

where A, B, C, D are real matrices of size $m\times m$, $m\times n$, $n\times m$, $n\times n$, respectively. In many real-life applications such as the transport theory related to the transmission of a particle beam [16] and the Markov process [3], the coefficient matrices in (1.1) and (1.2) constitute a block structure of M-matrix, that is,

$$K=\begin{pmatrix} D & -C \\ -B & A \end{pmatrix} \tag{1.3}$$

is a nonsingular M-matrix or an irreducible singular M-matrix. In this sense, we referred such equations as M-matrix algebraic Riccati equations (MAREs) [21]. The minimal nonnegative solution of the MARE is of great interest in real applications and its existence has been well studied in [15,16]. Lots of numerical iterative methods including the Newton's method and the fixed-point methods have been extensively studied to find the minimal nonnegative solution of MAREs, see [1, 2, 5, 7, 8, 10, 11, 13, 16, 21, 22] and the references therein. Among these methods, the structure-preserving doubling algorithm stands out for its quadratical convergence analogous to Newton's method and the faster convergence than fixed-point methods. With incorporating different matrix transformations, the doubling algorithm has various initial processes and they are respectively referred as the SDA [13], the SDA-ss [6] and the ADDA [21]. Concretely, for a matrix A, the SDA employs the Cayley transformation

$$\mathcal{C}(A,\alpha)=(A-\alpha I)(A+\alpha I)^{-1} \tag{1.4}$$

with $\alpha>0$. The SDA-ss admits the shift-and-shrink transformation

$$\mathcal{S}(A,\gamma)=I-A/\gamma \tag{1.5}$$

with $\gamma>0$ and the ADDA exploits the generalized Cayley transformation

$$\mathcal{G}(A,\beta,\alpha)=(A-\beta I)(A+\alpha I)^{-1} \tag{1.6}$$

with $\alpha>0$ and $\beta>0$. By selecting proper parameters α, β and γ, each of all above transformations is able to transfer n eigenvalues of Hamiltonian-like matrix

$$H=\begin{pmatrix} D & -C \\ B & -A \end{pmatrix} \tag{1.7}$$

on the left complex semi-plane to ones inside the unit circle and other n eigenvalues of H on the right complex semi-plane to ones outside the unit circle. Then each initial process of these doubling algorithms is well-defined and the whole iteration scheme is feasible.

In this paper, we still aim at the doubling algorithm for solving MAREs, but with a transformation different with (1.4), (1.5) and (1.6). The development story origins from