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Fracture Analysis in Orthotropic Thermoelasticity Using Extended Finite Element Method

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Abstract. In this paper, a method for extracting stress intensity factors (SIFs) in orthotropic thermoelasticity fracture by the extended finite element method (XFEM) and interaction integral method is present. The proposed method is utilized in linear elastic crack problems. The numerical results of the SIFs are presented and compared with those obtained using boundary element method (BEM). The good accordance among these two methods proves the applicability of the proposed approach and conforms its capability of efficiently extracting thermoelasticity fracture parameters in orthotropic material.

AMS subject classifications: 65M60, 74A45

Key words: Fracture, orthotropic, thermoelasticity, interaction integral method, XFEM.

1 Introduction

Recently, growing focus on orthotropic materials used in aerospace and automobile industries, temperature field problem may be involved in the engineer component, hence, studying thermal fracture in these elastic materials has been among the most interesting topics of research in recent decades. For simple and special geometry [1] problem, the analytic or semi-analytic method is frequently used. However, for general or complex geometries problem, numerical method is an advisable choice. The existing numerical methods such as finite element method whose meshes assignment require been conformed to the discontinuities or singular singularity. To overcome the difficulties, XFEM [2,3] has been proven to be an efficient method for discontinuities problems which

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does not need remeshing in the process of crack growth. For thermoelastic fracture problems using XFEM, thermal problems in [4–6] were involved to solve shear band problems with thermal effects in [7], the first paper about thermoelastic problems was discussed in [8]. In this paper, either adiabatic or isothermal condition is considered on the crack surface, the SIFs are extracted from the XFEM solution by an interaction integral, but it is only in isotropic materials. In [5], thermo-mechanical XFEM crack propagation analysis in functionally graded isotropic or orthotropic materials is dealt with. In [9], thermal and thermo-mechanical influence on crack propagation by extended meshless method is done. In [10], the thermal SIFs are obtained by crack closure integral or element-free Galerkin method. In [6, 9, 10], the material addressed is also limited to isotropic case, therefore, to my knowledge, for steady-state thermoelastic fracture problem in orthotropic medium using XFEM, few papers were reported in [5, 11].

In this paper, fracture analysis using XFEM in orthotropic thermoelastic problems is performed. The SIFs are extracted by interaction integral method. Several numerical examples are presented to validate the accuracy of results.

The outline of this paper is as follows. Section 2 recalls the fracture mechanics of orthotropic materials. Section 3 formulates the problem and introduces the discretization of the temperature field. In Section 4, the extraction of the SIFs from the XFEM solution is presented. It relies on interaction integrals in domain form with thermal effect. In Section 5, the method is illustrated by numerical examples and is compared with reference solutions. The conclusions are drawn in Section 6.

2 Fracture mechanics of orthotropic materials

The stress-strain relation [12] in linear elastic material can be written as

$$\varepsilon_\alpha = a_{\alpha\beta}\sigma_\beta, \quad (\alpha,\beta = 1,2,3), \tag{2.1}$$

with

$$\varepsilon_1=\varepsilon_{11}, \qquad \varepsilon_2=\varepsilon_{22}, \qquad \varepsilon_3=\varepsilon_{33}, \qquad \varepsilon_4=2\varepsilon_{23}, \qquad \varepsilon_5=2\varepsilon_{31}, \qquad \varepsilon_6=2\varepsilon_{12}, \tag{2.2a}$$

$$\sigma_1=\sigma_{11}, \qquad \sigma_2=\sigma_{22}, \qquad \sigma_3=\sigma_{33}, \qquad \sigma_4=\sigma_{23}, \qquad \sigma_5=\sigma_{31}, \qquad \sigma_6=\sigma_{12}, \tag{2.2b}$$

where $a_{\alpha\beta}$ is components of orthotropic compliance tensor. Here, we assume the material is orthotropic with any types of loadings or general boundary conditions and a crack. Let (X_1,X_2) be global Cartesian co-ordinate, (x,y) be local Cartesian co-ordinate and (r,θ) be local polar co-ordinate defined on crack tip. As shown in Fig. 1.

The characteristic equation for orthotropic materials can be obtained using equilibrium and compatibility conditions [12]

$$a_{11}\mu^4 - 2a_{16}\mu^3 + (2a_{12}+a_{66})\mu^2 - 2a_{26}\mu + a_{22} = 0. \tag{2.3}$$

The roots of Eq. (2.3) are always either complex or purely imaginary in conjugate pairs as μ_1, $\bar{\mu_1}$ and μ_2, $\bar{\mu_2}$.