Advances in Applied Mathematics and Mechanics
Adv. Appl. Math. Mech., Vol. **9**, No. 5, pp. 1145-1161

DOI: 10.4208/aamm.2015.m1195
October 2017

Application of Vectorial Wave Method in Free Vibration Analysis of Cylindrical Shells

R. Poultangari[1] and M. Nikkhah-Bahrami[2,*]

[1] *Department of Mechanical Engineering, Dezful Branch, Islamic Azad University, Dezful 64616-45165, Iran*
[2] *Department of Mechanical and Aerospace Engineering, Science and Research Branch, Islamic Azad University, Tehran 14515-775, Iran*

Received 9 July 2015; Accepted (in revised version) 13 October 2016

Abstract. The vectorial form of the Wave Propagation Method (VWM), regarding the dispersion of harmonic plain (elasto-dynamic) waves within certain wave-guides, is developed for the vibration analysis of circular cylindrical shells. To obtain this goal, all plain waves are divided into positive-negative going wave vectors along with the shell axis. Based on the Flügge thin shell theory, the shell continuity as well as boundary conditions are well satisfied by introducing the propagation and reflection matrices. Furthermore, all elements of the reflection matrix are derived for certain classical supports. As an example, for demonstrating the feasibility of VWM in the shell vibration analysis, a circular cylindrical shell with two ended flexible support is adopted. The natural frequencies of the system as well as mode shapes are obtained using VWM. The aquired results are compared with those of the previous works and found in excellent agreement. It is also found that VWM could mathematically provide a reduced dimensional matrix (dominant matrix) to calculate the natural frequencies of the system. Accordingly, the proposed method can provide high computational efficiency and remarkable accuracy, simultaneously.

AMS subject classifications: 70J30, 74K25

Key words: Elasto-dynamic waves, vectorial wave method, reflection matrices, circular cylindrical shell, Flügge theory, free vibration.

1 Introduction

Vibration of cylindrical shells, a critical industrial problem for many years, has long since come to the attention of scientific literature [1]. Presently, research in this area is underway to provide ways to achieve higher accuracy as well as higher computation speeds with minimum geometrical and mechanical limitations.

*Corresponding author.
Email: mbahrami@ut.ac.ir (M. N. Bahrami)

For free and forced vibration analysis in cylindrical shells, several ways have been proposed. Based on the cost of the required calculations for a specific accuracy, such methods can be divided into iterative and non-iterative methods. Each one of these methods could provide a special dominant matrix for the calculation of natural frequencies. Two important categories of the iterative methods are: numerical methods and close form solutions. The numerical methods based on the energy, weighted residual, differential quadrature and so on [2–4] give a large size dominant matrix (depending on the number of required frequencies) and hence require the largest amount of calculations. In the close form solutions [5, 6], as another sets of the iterative methods, depending on the complexity of the problem and the required accuracy, a converged solution can be obtained by using several analytical terms, leading to a large size dominant matrix. In a comparison, the non-iterative methods provide smaller size dominant matrices independent of the required accuracy and number of frequencies. Hence these methods are very fast and have a better convergency and accuracy [7].

It should be noted that vibration analysis of the elementary geometries such as beams, plates and more complex geometries such as cylindrical shells is a key for the analysis, control and optimization of more complex systems. Hence, any raise in the accuracy or any reduction in the computational cost of the solution method in vibration analysis of such elementary systems is of great interest, as it could open new approaches for the analysis of higher order or more complex systems, real time analysis, optimizations or inverse problems.

Furthermore, the low computational cost methods are of interest for monitoring, system failure detection as well as inverse problems in real time monitoring systems, in which deal with limited computational resources and system memories. From this point of view, any reduction in the required computational cost or memory of the solution approach is of great engineering and mathematical interest.

As far as shell vibrational problems are concerned, extensible non-iterative solutions generally include transfer function method, TFM (Laplace transform of shell Green's functions) [8] and wave propagation method (wave function expansion) [9–11]. Based on wave propagation method, two general techniques could be addressed for the analysis of shell vibrations in recent studies: state-space technique (SST) [12] and wave based method (WBM) [7].

The vectorial form of wave propagation method (VWM) can provide accurate and low cost non-iterative solutions for the vibration analysis of a system. This methodology has frequently been employed in the vibration analysis of non-shell mechanical elements, such as bars, beams and plates [13–19]. However, perhaps due to lake of relevant vector-matrix relations, the application of VWM in the vibration analysis of shells has not been addressed yet.

In the present study, VWM is developed for vibration analysis of circular cylindrical shells and the relevant matrix relations are introduced. Accordingly, using VWM, the positive-negative going wave vectors are defined and continuity and boundary conditions of the shell are satisfied by determining the propagation and reflection matrices. It