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Numerical Prediction of Natural Convection Flow in the Presence of Weak Magnetic Prandtl Number and Strong Magnetic Field with Algebraic Decay in Mainstream Velocity

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Abstract. In present work, we investigate numerical simulation of steady natural convection flow in the presence of weak magnetic Prandtl number and strong magnetic field by involving algebraic decay in mainstream velocity. Before passing to the numerical simulation, we formulate the set of boundary layer equations with the inclusion of the effects of algebraic decay velocity, aligned magnetic field and buoyant body force in the momentum equation. Later, finite difference method with primitive variable formulation is employed in the physical domain to compute the numerical solutions of the flow field. Graphical results for the velocity, temperature and transverse component of magnetic field as well as surface friction, rate of heat transfer and current density are presented and discussed. It is pertinent to mention that the simulation is performed for different values of algebraic decay parameter α, Prandtl number Pr, magnetic Prandtl number P_m and magnetic force parameter S.

AMS subject classifications: 65M10, 78A48

Key words: Natural convection, magnetized plate, skin friction, current density, heat transfer, algebraic decay in main stream velocity.

1 Introduction

The problem to be considered in this paper is that of laminar two dimensional boundary layer which is generated on vertical magnetized plate, when viscous, incompressible fluid flow over it in such a way that the free stream velocity outside the boundary layer by following Merkin [22] has the form $U_e(x)=(1-x)^{-\alpha}$. Goldstein [18] obtained similarity solutions of the boundary layer equations for (non-dimensional) mainstreams $U(x)$

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of the form $U(x)=(1-x)^{-\alpha}$, where x measures distance along the wall. Following Goldstein, Ackerberg [1] considered the steady, axisymmetric, converging motion of a viscous incompressible fluid inside an infinite circular cone and showed that there could be solutions with exponential decay for $\alpha\geq 1$, solutions with algebraic decay for $0<\alpha<1$ and no solutions for $\alpha\leq 0$. He concluded that the solutions with algebraic decay were unacceptable in the context of the usual boundary layer theory as they could not be matched with the outer expansion over a finite part of the x-axis.

Later on, Brownand Stewartson [8] pointed out that solutions of the boundary layer equations with algebraic decay could be allowed and would not contradict Goldstein's argument provided that they held only at singular points of the equations and not over a finite range of x. They showed that this type of similarity solution could be a limit of a solution of the full boundary layer equations as $x\to 1$ but that, in this case, this limit would not commute with the limit $y\to\infty$. A non-commutative limit of this type has also been discussed by Buckmaster [9] on the flow at the rear of a cylinder when separation has been completely suppressed by magnetohydrodynamical effects. Merkin [22] extended the work of Brown and Stewartson. He obtained a numerical solution of the boundary layer equations for mainstreams which are $\mathcal{O}((1-x)^{-\alpha})$ and derived a series expansion for the solution near $x=1$. Davies (see [13, 14]) examined the fact that the boundary layer thickness and drag coefficient decreases steadily as magnetic force parameter S increases. Tan and Wang [27] investigated that the rate of heat transfer decreases, the magnetic field and thermal boundary layer thicknesses increases with the increase of magnetic force parameter S. Hildyard [20] corrected the magnetic-field boundary condition used by Gribben and obtained the appropriate asymptotic solutions for large and small values of the magnetic Prandtl number P_{r_m}. Ingham [21] studied the boundary layer flow on a semi-infinite flat plate placed at zero incidence to a uniform stream of electrically conducting gas with an aligned magnetic field at large distances from the plate. Glauert [17] examined the basic steady flow in the case of uniform field in a magnetized plate. He showed that for both large and small ϵ, the conductivity parameter, separation occurs, when field strength parameter β reaches a critical value. The magnetohydrodynamic boundary layer flow past a magnetized plate when a uniform magnetic field in the stream direction is applied was studied by Chawala [10]. The case of uniform suction and blowing through an isothermal vertical wall was studied by Sparrow and Cess [25], he obtained a series solution which is valid near the leading edge. Later on, Merkin [23] followed the same problem in more detail and obtained asymptotic solutions, valid at large distances from the leading edge for both the suction and blowing. Clarke [11] solved boundary layer problem by using the method of matched asymptotic expansions numerically. The problem of blowing and wall temperature variations is discussed by Vedhanayagam et al. [29]. The case of heated isothermal horizontal surface with transpiration is studied by Clarke and Riley [12]. Gupta et al. [19] studied the hydromagnetic steady shear flow along an electrically insulating porous plate. Bikash and Sharma [7] observed the heat transfer characteristic from continuous flat surface of an electrically conducting fluid on non newtonian visco-elastic fluid. Zueco and Ahmed [30] obtained