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Exact Solutions for the Flow of Fractional Maxwell Fluid in Pipe-Like Domains

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Abstract. This paper presents an analysis of unsteady flow of incompressible fractional Maxwell fluid filled in the annular region between two infinite coaxial circular cylinders. The fluid motion is created by the inner cylinder that applies a longitudinal time-dependent shear stress and the outer cylinder that is moving at a constant velocity. The velocity field and shear stress are determined using the Laplace and finite Hankel transforms. Obtained solutions are presented in terms of the generalized G and R functions. We also obtain the solutions for ordinary Maxwell fluid and Newtonian fluid as special cases of generalized solutions. The influence of different parameters on the velocity field and shear stress are also presented using graphical illustration. Finally, a comparison is drawn between motions of fractional Maxwell fluid, ordinary Maxwell fluid and Newtonian fluid.

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Key words: Fractional Maxwell fluid, velocity field, shear stress, fractional calculus, Hankel transform, Laplace transform.

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1 Introduction

In continuum mechanics, a Newtonian fluid is a fluid in which the viscous stress arising from its flow is linearly proportional to the local strain rate. Water is one such example of Newtonian fluid. Newtonian fluids are the simplest mathematical models of fluids that account for viscosity.

On the other hand, lots of fluids practically do not follow this linear relationship between the shear stress and the rate of strain. These fluids are called non-Newtonian fluids. Examples of such fluids are saliva, jam, ketchup, synovial fluid, mayonnaise, lava etc. The non-Newtonian fluids are very important in practice and have many applications in industrial and engineering fields. These fluids are very frequently encountered in many

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different fields such as food industries, chemical engineering, biomedicine etc. and also are relevant to many other industrial processes. Hence, study of flow of non-Newtonian fluids has become a subject of great importance. Classical Navier-Stokes theory is sufficient to analyze the flow behavior of Newtonian fluids but it cannot describe the flow of non-Newtonian fluids due to the nonlinear viscoelastic behavior.

There are many models that have been proposed for non-Newtonian fluids such as rate type [1], differential type [2] etc. Among them, the rate type model is the most popular. Differential type model does not describe the influence of relaxation and retardation times and also cannot describe the flow of some polymers.

The first exact solutions for a second grade non-Newtonian fluid in cylindrical domain were proposed by Ting [3]. Srivastava [4] and Waters & King [5] proposed the same for Maxwell and Oldroyd-B fluids respectively. Similarly, the exact solutions for motion of non-Newtonian fluids due to shear stress were proposed by Bandelli & Rajagopal [6]. Bandelli et al. [7] and Waters & King [8] proposed the same for second-grade and Oldroyd-*B* fluids respectively. Recently, a lot of papers have been published regarding such fluid motions [9–16].

Fractional calculus has been widely used to describe viscoelastic behavior of fluids [17, 18]. For non-Newtonian fluids, the time derivative of an integer order in a classical differential equation is replaced by the Riemann-Liouville fractional calculus operator. We can define non-integer order derivatives or integrals using this operator [19]. Fang et al. [20] published their work on the Rayleigh-Stokes problem for a heated generalized second grade fluid using fractional derivative model. Recently, many published papers have used the fractional calculus for work done on non-Newtonian fluids [21–25].

The aim of this paper is to provide the exact solutions for the velocity field and shear stress corresponding to the unsteady flow of an incompressible fractional Maxwell fluid in annular region of cylindrical domain. At time $t=0^+$, the inner cylinder is pulled with a time-dependent shear stress and the outer cylinder is moving at a constant velocity. The solution is obtained using Laplace and Hankel transform methods and the results are presented in terms of generalized G and R functions. The solutions for ordinary Maxwell and Newtonian fluids are also obtained as limiting cases of $\alpha\to 1$ and $\alpha\to 1$, $\lambda\to 0$, respectively.

2 Governing equations

Consider an incompressible fractional Maxwell fluid that has a velocity V and extra shear stress S of the form [26]

$$V=V(r,t)=v(r,t)e_z, \quad S=S(r,t), \tag{2.1}$$

where e_z is the unit vector in the z direction of the cylindrical coordinates. For such flows, the constraint of incompressibility is automatically satisfied, while the governing