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Elastic-Plastic Analysis for Functionally Graded Thick-Walled Tube Subjected to Internal Pressure

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Abstract. The elastic-plastic response of the functionally graded thick-walled tube subjected to internal pressure is investigated by using the relation of the volume average stresses of constituents and the macroscopic stress of composite material in micromechanics. The tube consists of two idealized isotropic elastic-plastic materials whose volume fractions are power functions of the radius. As the internal pressure increases, the deformations of one phase and two phases from elastic to plastic are analyzed. In order to simplify the calculations we assume both materials with the same Poisson's ratio. By using the assumption of a uniform strain field within the representative volume element and the Tresca yield criterion, the theoretical solutions are obtained for the case of two elastic phases and the case of two plastic phases, and the function of the radial displacement is presented for the case with both elastic and plastic phases. The yield criterion of functionally graded material is given in terms of the yield stresses and volume fractions of constituents rather than Young's modulus and yield stress with different unknown parameters of the whole material in the existing papers. Finally we also discuss the position where the plastic deformation first occurs and the conditions for which material first yields in the tube.

AMS subject classifications: 74B05, 74E30, 74M25

Key words: Functionally graded thick-walled tube, internal pressure, elastic-plastic solution.

1 Introduction

Functionally graded materials (FGMs) are composite materials formed of two or more constituent phases with a continuously variable composition. FGMs have a lot of advantages that make them attractive in potential applications, including a potential reduction of in-plane and transverse through-the-thickness stresses, an improved residual stress

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distribution, enhanced thermal properties, higher fracture toughness, and reduced stress intensity factors [1, 2]. Recently the FGMs are widely used for structures subjected to thermal and mechanical loading, and the mechanical behaviors of these FGMs have been studied by using some numerical computation methods [3–5].

A sufficiently long tube subjected to internal pressure is one of the classical problems in engineering mechanics. In the purely elastic domain, many researchers have explored the mechanical behaviors of the FGM long tube such as pressure vessels and cylinders by assuming different kinds of functions of Young's modulus [6–17]. For example, Young's modulus $E=E_0e^{\xi r}$ (E_0 and ξ are material constants, r is the radial coordinate of the cylinder) is an exponential function of the radius [6], $E=E_0r^{\beta}$ (E_0 and β are material constants) is a power function of the radius [7] and $E=A+Br$ (A and B are material constants) is a linear function of the radius [8].

Elastic-plastic analyses of thick-walled pressure vessels have attracted a lot of interest due to their important applications in engineering. The analytical solutions of stresses and radial displacement are given for idealized elastic-plastic [18] and work-hardening [19] homogeneous materials. Recently, the elastic-plastic deformations of FG-M spherical pressure vessels are studied in the idealized elastic-plastic material with the Tresca yield criterion $\sigma_\theta-\sigma_r=\sigma_0$, where σ_0 is either a material constant [20,21] or a power function of the radius $\sigma_0(r)=\sigma_C(r/b)^m$ (σ_C and m are material constants) [22–25].

Many literatures mentioned above focus on the assumption that Young's modulus and the yield stress vary gradient in the tube. In most actual systems, several FGMs are manufactured by two phases of materials with different properties. Generally, the overall elastic modulus can be found in terms of their properties of constituents and the volume fractions in a certain regulation [26–28]. However, the definition of the yield stress for the FGM tube is complicated due to different elastic-plastic behavior of materials used as constituents of the FGM tube. Due to different material properties of the constituents of the FGM tube, different deformations and stresses of the constituents will occur for the tube subjected to the increasing internal pressure, which may lead to the constituents yielding in a certain sequence. In particular, for the FGM tube consists of two materials, both two materials are in elastic domain with small internal pressure and then one phase yields first at the inner radius when its stresses satisfy the plastic yield condition while the other phase is still in elastic state. With the continue increasing of internal pressure, the other phase also yields to plastic deformation at the inner radius, then the tube from the inner to the outer radius can be divided into three regions: fully plastic region, partially plastic region (only material B is plastic) and purely elastic region. Detailed discussion is shown in the following.

In this paper, we first define a volume fraction varied radially rather than the previous literature's [6–17,20–25] assumption of Young's modulus and the yield stress of the FGM tube. The tube consists of two idealized isotropic elastic-plastic materials whose volume fractions are power functions of the radius. In Section 2, the basic equations of the FGM long tube are first presented. Section 3 analyzes the elastic and plastic mechanical behaviors of the tube, including the elastic, the elastic-plastic and the plastic deformations.