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Solution of the Magnetohydrodynamics Jeffery-Hamel Flow Equations by the Modified Adomian Decomposition Method

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Abstract. In this paper, the nonlinear boundary value problem (BVP) for the Jeffery-Hamel flow equations taking into consideration the magnetohydrodynamics (MHD) effects is solved by using the modified Adomian decomposition method. We first transform the original two-dimensional MHD Jeffery-Hamel problem into an equivalent third-order BVP, then solve by the modified Adomian decomposition method for analytical approximations. Ultimately, the effects of Reynolds number and Hartmann number are discussed.

AMS subject classifications: 76W05, 34B15

Key words: Jeffery-Hamel flow, magnetohydrodynamics, nonlinear differential equation, Adomian decomposition method, Adomian polynomials.

1 Introduction

Jeffery [1] and Hamel [2] have worked on incompressible viscous fluid flow through convergent-divergent channels, mathematically. They presented an exact similarity solution of the Navier-Stokes equations. The special case of two dimensional flow through a channel with inclined plane walls meeting at a vertex and with a source or sink at the vertex has been studied by several authors [3–7].

Most scientific problems such as Jeffery-Hamel flows and other fluid mechanic problems are inherently in form of nonlinearity. Except a limited number of these problems, most of them do not have exact solution. Therefore, these nonlinear equations should be solved using other methods. Therefore, many different methods have been introduced to obtain analytical approximate solutions for these nonlinear problems, such as the perturbation method [8, 9], orthogonal polynomial and wavelet methods [10], methods of

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travelling wave solutions [11], the Adomian decomposition method (ADM) and the variational iteration method [12].

One of the most applicable analytical techniques is the ADM [13–22]. It is a practical technique for solving nonlinear functional equations, including ordinary differential equations, partial differential equations, integral equations, integro-differential equations, etc. The ADM provides efficient algorithms for analytic approximate solutions and numeric simulations for real-world applications in the applied sciences and engineering without unphysical restrictive assumptions such as required by linearization and perturbation. The accuracy of the analytic approximate solutions obtained can be verified by direct substitution.

In the ADM, the solution $u(x)$ is represented by a decomposition series

$$u(x)=\sum_{n=0}^{\infty}u_n(x), \tag{1.1}$$

and the nonlinearity comprises the Adomian polynomials

$$Nu(x)=\sum_{n=0}^{\infty}A_n(x), \tag{1.2}$$

where the Adomian polynomials $A_n(x)$ is defined for the nonlinearity $Nu=f(u)$ as [20]

$$A_n(x)=A_n(u_0,u_1,\cdots,u_n)=\frac{1}{n!}\frac{\partial^n}{\partial\lambda^n}f\Big(\sum_{k=0}^{\infty}\lambda^k u_k(x)\Big)\Big|_{\lambda=0}. \tag{1.3}$$

Different algorithms for the Adomian polynomials have been developed by Rach [23,24], Wazwaz [25], Abdelwahid [26] and several others [27–30]. Recently new algorithms and subroutines in MATHEMATICA for fast generation of the Adomian polynomials to high orders have been developed by Duan [31–33].

The solution components are determined by recursion scheme. The nth-stage approximation is given as $\phi_n(x)=\sum_{k=0}^{n-1}u_k(x)$.

We remark that the convergence of the Adomian series has already been proven by several investigators [23,34–36]. For example, Abdelrazec and Pelinovsky [36] have published a rigorous proof of convergence for the ADM under the aegis of the Cauchy-Kovalevskaya theorem. In point of fact the Adomian decomposition series is found to be a computationally advantageous rearrangement of the Banach-space analog of the Taylor expansion series about the initial solution component function.

2 Mathematical formulation

We present model of the MHD Jeffery-Hamel flow problem, which describes an exact similarity solution of the Navier-Stokes equations for special case of two-dimensional flow through a channel with inclined plane walls.