Advances in Applied Mathematics and Mechanics
Adv. Appl. Math. Mech., Vol. **7**, No. 5, pp. 644-662

DOI: 10.4208/aamm.2014.m632
October 2015

Thermo-Solutal Natural Convection in an Anisotropic Porous Enclosure Due to Non-Uniform Temperature and Concentration at Bottom Wall

Ashok Kumar*, Pravez Alam and Prachi Fartyal

Department of Mathematics, HNB Garhwal University (A Central University), Srinagar–246174, Uttarakhand India

Received 26 May 2014; Accepted (in revised version) 3 November 2014

Abstract. This article summaries a numerical study of thermo-solutal natural convection in a square cavity filled with anisotropic porous medium. The side walls of the cavity are maintained at constant temperatures and concentrations, whereas bottom wall is a function of non-uniform (sinusoidal) temperature and concentration. The non-Darcy Brinkmann model is considered. The governing equations are solved numerically by spectral element method using the vorticity-stream-function approach. The controlling parameters for present study are Darcy number (Da), heat source intensity i.e., thermal Rayleigh number (Ra), permeability ratio (K^*), orientation angle (ϕ). The main attention is given to understand the impact of anisotropy parameters on average rates of heat transfer (bottom, Nu_b, side Nu_s) and mass transfer (bottom, Sh_b, side, Sh_s) as well as on streamlines, isotherms and iso-concentration. Numerical results show that, for irrespective value of K^*, the heat and mass transfer rates are negligible for $10^{-7} \le Da \le 10^{-5}$, $Ra = 2\times 10^5$ and $\phi = 45°$. However a significant impact appears on Nusselt and Sherwood numbers when Da lies between 10^{-5} to 10^{-4}. The maximum bottom heat and mass transfer rates (Nu_b, Su_b) is attained at $\phi = 45°$, when $K^* = 0.5$ and 2.0. Furthermore, both heat and mass transfer rates increase on increasing Rayleigh number (Ra) for all the values of K^*. Overall, It is concluded from the above study that due to anisotropic permeability the flow dynamics becomes complex.

AMS subject classifications: 65M10, 78A48

Key words: Spectral element method, anisotropic porous medium, non-Darcy Brinkmann model, non-uniform heating.

1 Introduction

In many natural and technological processes, temperature and mass or concentration diffusion act together to induce a buoyancy force which drives the fluid, and it is known

*Corresponding author.
Email: ashrsdma@gmail.com (A. Kumar)

as double-diffusive or thermo-solutal convection. The basic feature of double-diffusive convection is that two components with different rates of diffusion affect the fluid density. In oceanography [1], convection processes involve thermal and salinity gradients. Double-diffusive convection is more complex than the single diffusive convective flow. From practical and industrial point of view, natural convection due to combined buoyancy effect of thermal and species diffusion in a fluid saturated porous medium is received considerable attention. The migration of moisture in fibrous insulation, contaminants transport in the environment, solidification of binary alloy and crystal growth, melting and cooling near ice surfaces, sea water intrusion into freshwater lakes, dispersion of dissolvent materials or particulate matter in flows etc. are few examples, where double-diffusive convection is involved. The recent comprehensive literature review on thermo-solutal convection in porous media has been conducted by Nield and Bejan [2].

In the above type of configuration generally two situations buoyancy assisted and buoyancy opposed are considered. Several experimental and numerically studies in this field (without porous medium) are reported by Kamotani et al. [3], Lee et al. [4], Beghein et al. [5], Bennacer and Gobin (see [6, 7]), and Ghorayeb and Mojtabi [8] etc. Double-diffusive natural convection in a cubic enclosure subject to opposing and horizontal gradients of heat and solut is studied by Sezai and Mohamad [9]. Natural convection with combine heat and mass transfer in a vertical slot filled with isotropic porous medium, whose vertical walls are subjected to uniform fluxes of heat and mass, while the horizontal walls are insulated and impermeable, is analyzed by Trevisan and Bejan [10], Mehta and Nandkishor [11] and Mamou et al. (see [12, 13]) etc.

A comprehensive studies on thermo-solutal natural convection in a rectangular cavity filled with isotropic porous medium where different but uniform temperature and concentration are specified at two vertical walls and horizontal walls are adiabatic, are analyzed by Trevisan and Bejan [14], Goyeau et al. [15], and Kramer et al. [16]. Alavyoon [17] has reported the existence of analytical solution for slender cavity, when flow is induced by heat and mass fluxes on the vertical walls. Later on, for opposing heat and mass fluxes on the vertical walls, flow oscillation was reported by Alavyoon et al. [18].

Similar type of problems have also been investigated to understand anisotropic behavior of the medium in double-diffusive natural convection by Nitiarasu et al. [19], Bera et al. [20–22]) and Bennacer et al. [23] etc. Nithiarasu et al. [19] have studied the double-diffusive natural convective flow within a rectangular enclosure using a non-Darcy generalized model. They have reported that non-Darcy effects on flow, heat and mass transfer are significant, when the Rayleigh or Darcy numbers are large. Bera et al. [20] have presented significant effect of orientation angle as well as anisotropic permeability on flow rate, and on the overall heat and mass transfer rates. A numerical and analytical investigation on double diffusive natural convection, induced by buoyancy forces along with hydrodynamic, thermal, and solute anisotropy, is examined by Bera et al. [21]. They have shown the significant effect of anisotropic orientation angles of thermal and solute diffusivity on the overall heat and mass transfer rates. Later on, this investigation is extended by Bera and Khalili [22] and reported that, the existence of multiple steady so-