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Local RBFs Based Collocation Methods for Unsteady Navier-Stokes Equations

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Abstract. The local RBFs based collocation methods (LRBFCM) is presented to solve two-dimensional incompressible Navier-Stokes equations. In avoiding the ill-conditioned problem, the weight coefficients of linear combination with respect to the function values and its derivatives can be obtained by solving low-order linear systems within local supporting domain. Then, we reformulate local matrix in the global and sparse matrix. The obtained large sparse linear systems can be directly solved instead of using more complicated iterative method. The numerical experiments have shown that the developed LRBFCM is suitable for solving the incompressible Navier-Stokes equations with high accuracy and efficiency.

AMS subject classifications: 65M10, 35L03

Key words: Navier-Stokes equations, local collocation method, particular solutions, radial basis functions (RBFs), large sparse linear systems.

1 Introduction

Due to avoid the high cost of mesh generation, the interest in meshless methods to solve partial differential equations (PDEs) has generated considerably in the past decades. In contrary to the mesh based methods like the the finite difference method (FDM), the finite volume method (FVM) and the finite element method (FEM), meshless methods use a set of uniform or random points to discrete derivatives. Those points are not necessarily interconnected in the form of a grid. Among them a class of meshless methods based on radial basis functions (RBFs) seem much more appealing and have been considered

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as a prospective numerical methods for solving partial differential equations (PDEs). Originally, RBFs were proposed for scattered data and function interpolation by Hardy [1]. However, due to the fact that methods based on RBFs are truly meshfree and spatial dimension independent, which can easily be applied to solve high dimensional problems, researchers got more and more interested in the implementation of RBFs in PDEs solvers. The first trial was successfully done by Kansa [2,3]. Franke [4] showed that Hardy's multiquadric (MQ) and Duchon's thin-plate spline (TPS) ranked the best in accuracy, efficiency, and stability. Recently, an increasing number of researchers extended RBFs' methods to solve various ordinary and partial differential equations including regularized long wave (RLW) equation [5], Hirota-Satsuma coupled KdV equations [6], the solution of 2D biharmonic equations [7], the case of heat transfer equations [8], incompressible Navier-Stokes equations [9,10], natural convection problems [11], elliptic problems with variable coefficients [12,13], scalar wave equations [14], natural convection problems, hyperbolic partial differential equations [15] and so on.

In general, the RBFs used in the collocation technique are globally supported. The resultant matrices are often dense and even ill-conditioned, and thus restricting our ability to solve large-scale problems. To circumvent this issue, considerable efforts have been made such as domain decomposition [17], multigrid approach and compactly supported RBFs [18], the improved truncated singular valued decomposition [19], etc. However, among all those dealings localized methods are more direct and simple. Currently many localized meshless methods have been developed. For example, Shu et al. [9,10] applied the local RBF-based differential quadrature (LRBFDQ) method to simulate incompressible Navier-Stokes equations in 2003 and then Lee et al. [20], Sarler et al. [21] proposed local radial basis function collocation method on the basis of global RBFCM and applied it to solve convective-diffusive problems. Recently, Chen et al. [16] presented the localized method of approximated particular solutions (LMAPS). By adopting those methods, we can successfully avoid severe ill-conditioning problem and form a sparse coefficient matrix as approximations by RBF interpolation are all conducted within local domain. In this paper, motivated by the idea of local RBFs based meshless method, we will present readers the so-called local radial basis function collocation method (LRBFCM), and apply it to solve two-dimensional, time-dependent, incompressible Navier-Stokes equations in vorticity-stream function form.

The organization of the paper is as follows. In Section 2, the formulations of the LRBFCM are listed for solving two-dimensional, time-dependent Navier-Stokes equations in vorticity-stream function form in detail. In Section 3, numerical examples are given to demonstrate the effectiveness of the proposed procedure. In Section 4, we draw some conclusions.

2 Formulation of the LRBFCM for incompressible N-S equations