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Piecewise Constant Level Set Algorithm for an Inverse Elliptic Problem in Nonlinear Electromagnetism

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Abstract. An inverse problem of identifying inhomogeneity or crack in the workpiece made of nonlinear magnetic material is investigated. To recover the shape from the local measurements, a piecewise constant level set algorithm is proposed. By means of the Lagrangian multiplier method, we derive the first variation w.r.t the piecewise constant level set function and obtain the descent direction by the adjoint variable method. Numerical results show the robustness and effectiveness of our algorithm applied to reconstruct some complex shapes.

AMS subject classifications: 65K10, 35Q60, 74M10

Key words: Nonlinear electromagnetism, adjoint variable method, shape reconstruction, piecewise constant level set algorithm.

1 Introduction

In many applications, one needs to find out the flaws in materials nondestructively. Inspired by this, the non-destructive evaluation technique has attracted the eyes of many researchers during the last decades. As one kind of non-destructive evaluation technique, eddy current testing technique [1] has been used for flaw detection. In this paper, we intend to design an algorithm to identify the crack or inhomogeneities in the nonlinear magnetic material like steel from the local measurements of the magnetic induction.

This inverse problem is a problem of shape reconstruction. Similar to general shape recovery problem, there is no information about the interface of the optimal shape as a prior, so we need to have a good mechanism to express the shape and track the evolution of the shape. The level set method was first originally proposed by Osher and Sethian in [2]. For this method, the interface between two adjacent domains is represented by

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the zero level set of a Lipschitz continuous function. Through the change of this function, this method can easily handle many types of shape and topological changes, such as merging, splitting and developing sharp corners. Due to these merits, it has been used in various areas, such as epitaxial growth [3], inverse problem, optimal design [4], image segmentation [5], structure topology optimization [6] and EIT problem [7]. For the seek of the numerical stability, the level set function is usually chosen as the signed distance function, but at most cases, the level set function after each iteration is not be a signed distance function and is usually re-initialized by solving an ordinary differential equation [8]. In [9], Cimrák et al. used the level set method to represent the shape of the inhomogeneity and evolve the shape by minimizing a functional during the iterative process. In [10, 11], Cimrák also used the level set method for the representation of the interface to solve some inverse problems in thermal imaging and the nonlinear ferromagnetic material. As to the initial value of the level set function, it was reported in [12–14] that the level set method based only on the shape sensitivity may get stuck at shapes with fewer holes than the optimal geometry in some applications such as structure designs. When one wants to use the level set method to solve the practical problem, he can reduce the effects of the initial value of the level set function on the final results in the next two ways. The first way is to choose the shape with enough holes as the initial value. The second one is to introduce the topological derivative into the level set method to let the shape create holes in the iterations [14–18]. To find a good initial value of the level set function, the researchers in [9] proposed the gradient-for-initial approach which is based on the idea that the domain which can drop the value of the cost functional should be the air gap. In that approach, the parameter in smeared-out Heaviside function should be set large enough. The large value of this parameter, however, can cause the oscillation phenomenon. So the parameter in the acquisition of the initial choice of the level set function and the evolving process of level set function should be set separately

Recently, piecewise constant level set method, which is a variant of level set method, was proposed by Lie, Lysaker and Tai in [20–22]. To distinguish these two methods, we call the former one traditional level set method. Unlike the traditional level set method, the interface between two adjacent sub-domains is represented by the discontinuity of a piecewise constant level set function. Compared with traditional level set method, piecewise constant level set method has at least two advantages. One merit is that it can create many small holes automatically without the topological derivatives during the iterative process. Furthermore, it is verified by many numerical examples that the final result is independent of the initial value of the level set function in many numerical tests. And the other one is that the piecewise constant level set method need not to re-initialize the level set function periodically during the evolution process, thus, reduces the computational cost a lot. Since it was proposed, it has been applied in various fields such as image segmentation, elliptic inverse coefficient identification, optimal shape design, electrical impedance tomography and positron emission tomography [19]. Lie, Lysaker and Tai took this method to solve the image segmentation in [20–23] and the elliptic inverse problem and interface motion problem in [24, 25]. Wei and Wang used piecewise