Advances in Applied Mathematics and Mechanics
Adv. Appl. Math. Mech., Vol. **4**, No. 2, pp. 205-222

DOI: 10.4208/aamm.10-m11104
April 2012

Semi-Analytical Solution for Functionally Graded Solid Circular and Annular Plates Resting on Elastic Foundations Subjected to Axisymmetric Transverse Loading

A. Behravan Rad*

Department of Mechanical Engineering, Karaj Branch, Islamic Azad University, Karaj, Iran

Received 26 May 2011; Accepted (in revised version) 30 November 2011

Available online 26 March 2012

Abstract. In this paper, the static analysis of functionally graded (FG) circular plates resting on linear elastic foundation with various edge conditions is carried out by using a semi-analytical approach. The governing differential equations are derived based on the three dimensional theory of elasticity and assuming that the mechanical properties of the material vary exponentially along the thickness direction and Poisson's ratio remains constant. The solution is obtained by employing the state space method (SSM) to express exactly the plate behavior along the graded direction and the one dimensional differential quadrature method (DQM) to approximate the radial variations of the parameters. The effects of different parameters (e.g., material property gradient index, elastic foundation coefficients, the surfaces conditions (hard or soft surface of the plate on foundation), plate geometric parameters and edges condition) on the deformation and stress distributions of the FG circular plates are investigated.

AMS subject classifications: 15A16, 15A18, 65D32, 74B05, 74G15

Key words: Functionally graded circular plate, elastic foundation, differential quadrature method, state-space method.

1 Introduction

Functionally graded materials (FGMs) have gained considerable attention in recent years. FGMs are a new kind of composite materials with wide range of applications. Since their material properties vary as a function with respect to the coordinates, their problems are more complicated than those of the homogeneous materials. FGMs are

*Corresponding author.
Email: behravanrad@gmail.com (A. B. Rad), abehravanrad@aol.com (A. B. Rad)

composite materials that are microscopically inhomogeneous, and their mechanical properties vary continuously in one (or more) direction(s). This is achieved by gradually changing the composition of the constituent materials along one direction, usually in the thickness direction, to obtain smooth variation of material properties and optimum response to the externally applied loading. The static and dynamic analysis of the FGM structural components is important in the design stage. Several researchers can be found in the literature in the field of structural analysis of FGM components using different methods. For example, Chen et al. [1] applied the DQ method to the analysis of geometrically nonlinear vibration of immovably simply supported beams. Reddy et al. [2] investigated axisymmetric bending of an FGM circular plate based on the first-order plate theory and obtained the relationships between the first-order plate theory and the classical thin plate theory. Chen et al. [3] applied the Hadamard and SJT matrices product with differential quadrature (DQ) rule to solution of geometrically nonlinear bending of isotropic and orthotropic rectangular plates. Yang and Shen [4] dealt with the dynamic response of initially stressed functionally graded rectangular thin plates subjected to partially distribute impulsive lateral loads. Ma and Wang [5] studied the axisymmetric bending of an FGM circular plate with the third-order plate theory. Vel and Batra [6] presented a three-dimensional exact solution for free and forced vibrations of simply supported functionally graded rectangular plates. Chen [7] investigated the nonlinear vibration of functionally graded plates with arbitrary initial stresses, effects of the amplitude of vibration, initial conditions and volume fraction on nonlinear vibration were studied. Serge [8] considered the problems of free vibrations, buckling, and static deflections of functionally graded plates whose material properties vary through the thickness. Park and Kim [9] analyzed the thermal post buckling and vibration of the functionally graded plates considering nonlinear temperature-dependent material properties. Nie and Zhong [10] investigated the bending of two-directional FG circular and annular plates based on the three-dimensional theory of elasticity using the state- space method combined with the DQM. Three-dimensional free and forced vibration analysis of functionally graded circular plates with material properties that vary continuously in the thickness direction and various boundary conditions was presented by Nie and Zhong [11]. Li et al. [12] presented the elasticity solutions for a transversely isotropic FGM circular plate subject to an axisymmetric transverse load in terms of the polynomials of even order. Free and forced vibration analysis of functionally graded annular sectorial plates with simply supported radial edges and arbitrary circular edges was carried out by Nie and Zhong [13]. Huang et al. [14] presented an exact solution for FG rectangular thick plates resting on elastic foundations, based on the three-dimensional theory of elasticity, using infinite double series of trigonometric functions combined with the state- space method. Wang et al. [15] applied the direct displacement method to investigate the free axisymmetric vibration of the transversely isotropic circular plates. Malekzadeh [16] used the DQ method to analysis the free vibration of thick FG rectangular plates supported by two-parameter elastic foundations. Hosseini-Hashemi et al. [17] investigated buckling and free vibration behaviors of radially functionally