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Variation of Parameters Method for Solving System of Nonlinear Volterra Integro-Differential Equations

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Abstract. It is well known that nonlinear integro-differential equations play vital role in modeling of many physical processes, such as nano-hydrodynamics, drop wise condensation, oceanography, earthquake and wind ripple in desert. Inspired and motivated by these facts, we use the variation of parameters method for solving system of nonlinear Volterra integro-differential equations. The proposed technique is applied without any discretization, perturbation, transformation, restrictive assumptions and is free from Adomian's polynomials. Several examples are given to verify the reliability and efficiency of the proposed technique.

AMS subject classifications: 65N99, 65R20, 65K15

Key words: Variation of parameters method, oceanography, system of nonlinear Volterra integro-differential equations, error estimates.

1 Introduction

It is well-known fact that a wide class of problems in physical and engineering sciences including oceanography, nano-hydrodynamics, drop wise condensation, glass-forming process, and wind ripple in desert can be studied in the general and unified framework of integro-differential equations, see [1–6, 14–33] and the references therein. It has been shown in [7] that fractional order integro-differential equations can be used to model nonlinear oscillations of earthquake. Oceanography is the study of the ocean making use of the various sciences including physics, chemistry, biology, gelogy and mathematics. Physical studies are carried out both by direct observation

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of the properties and movements and also by applying the basic physical principles of mechanics and thermodynamics to determine the motion. The observational approach is known as descriptive or geomathematical oceanography. The dynamical oceanography is used to endeavor to obtain mathematical relations between the forces acting on the ocean water and their consequent motions.

Due to the importance of nonlinear integro-differential equations, several numerical and analytic techniques including modified Adomian's decomposition method [31], Adomian's decomposition method [3], rationalized Haar functions method [14], homotopy peturbation method [1, 4, 5, 19], variational iteration method [15–17, 24–26] and modified variation of parameters method [20] have been developed. Wazwaz [32] used Modified Adomian's decomposition method to solve some integro-differential equations related to Blasius problems. Sayed et al. [3] applied decomposition method to solve linear Voltera Fredholm integro-differential equations. Maleknejad et al. [14] solved system of linear integro-differential equations by using rationalized Haar functions method and Biazar et al. [1] applied homotoppy perturbation method to solve nonlinear system of integro-differential equations. Ghasemi et al. [5] and Yusufoglu [34] used homotopy perturbation method for solving Volterra integro-differential equations. Wang et. al. [31] and Nadjafi et al. [18] applied variational iteration method to solve system of nonlinear integro-differential equations. Mohyud-Din et al. [16] have solved nonlinear system of integro-differential equations by modified variation of parameters method in which he coupled both homotopy perturbation method and variation of parameters method. Most of these methods have their inbuilt deficiencies like calculation of Adomian's polynomials, use of small parameters, identification of Lagrange multiplier, divergent results and huge computational work. These facts motivated us to consider variation of parameters method [8–10, 15–17, 24–26] for solving system of nonlinear integro-differential equations. This technique is a very useful tool in analytic studies and helps to improve our understanding of what dynamical effects may be important. The use of multiplier in variation of parameters method increase the rate of convergence by reducing the number of iterations, reduce the successive applications of integral operator and make the solution procedure simple while still maintaining a very high level of accuracy. The multiplier used in variation of parameters method is obtained by Wronskian technique and is totally different from Lagrange multiplier of variational iteration method.

Moreover, variation of parameters method removes the higher order derivative term from its iterative scheme which is clear advantage over the variational iteration method as the term may cause of repeated computation and calculations of unneeded terms, which consumes both the time and effort, in most of the cases. Thus variation of parameters method has reduced lot of computational work involved due to this term as compared to some other existing techniques using this term which is clear advantage of proposed technique over them. Hence, variation of parameters method provides a wider and better applicability as compare to other classical techniques. Ma et al. [8–10] presented variation of parameters method to solve some nonhomogenous partial differential equations. Ramos [28] used variation of parameters method to find