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# An Iterative Two-Grid Method of A Finite Element PML Approximation for the Two Dimensional Maxwell Problem

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**Abstract.** In this paper, we propose an iterative two-grid method for the edge finite element discretizations (a saddle-point system) of Perfectly Matched Layer(PML) equations to the Maxwell scattering problem in two dimensions. Firstly, we use a fine space to solve a discrete saddle-point system of $H(grad)$ variational problems, denoted by auxiliary system 1. Secondly, we use a coarse space to solve the original saddle-point system. Then, we use a fine space again to solve a discrete $\boldsymbol{H}(curl)$-elliptic variational problems, denoted by auxiliary system 2. Furthermore, we develop a regularization diagonal block preconditioner for auxiliary system 1 and use $H$-$X$ preconditioner for auxiliary system 2. Hence we essentially transform the original problem in a fine space to a corresponding (but much smaller) problem on a coarse space, due to the fact that the above two preconditioners are efficient and stable. Compared with some existing iterative methods for solving saddle-point systems, such as PMinres, numerical experiments show the competitive performance of our iterative two-grid method.

**AMS subject classifications**: 65F10, 65N30, 78A46
**Key words**: Maxwell scattering, edge finite element, PML, iterative two-grid method.

## 1 Introduction

An early paper of Bérenger [3] proposed a perfectly matched layer (PML) method for

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time-dependent Maxwell equations. The idea is to construct a fictitious absorbing layer outside the "region of interest" so that plane waves passed into the layer without reflection. This approach was then applied to various time domain problems (cf. [1, 4, 8, 9]). PML methods were also developed in terms of a complex change of variable (or stretching) for frequency domain Maxwell problems (cf. [5, 10, 17]). Especially, Bramble and Pasciak [5] have proved existence and uniqueness of the solutions to the infinite domain and truncated domain PML equations provided that the truncated domain is sufficiently large. Furthermore, they also showed the PML reformulation preserves the solution in the layer while decaying exponentially outside of the layer. However, the corresponding edge finite element discretization is an indefinite saddle-point system which is usually large and higher ill-condition. Hence constructing the corresponding fast algorithms is necessary for realistic computational electromagnetism.

Nowadays, there are only few research results for fast algorithms of PML equations. For example, Botros and Volakis [6] presented a generalized minimal residual (GMRES) solver which coupled with an approximate inverse preconditioner. Botros and Volakis [7] given an optimal selection of the PML parameters and tested the GMRES solver.

The two-grid methods are developed originally for nonsymmetric or indefinite linear elliptic partial differential equations (PDEs) [15, 18–20], and the basic idea is first to solve the original problem in a coarse mesh space with mesh size $H$ and then solve a symmetric positive definite (SPD) problem on a fine mesh space with mesh size $h$. This method was later extended to other problems (cf. [13,16,21,22]). However, the extension of the two grid method to the Maxwell equations is not straightforward, since the leading term *curl* for Maxwell's equations has a large kernel. Noting that the behaviors of the system PML problems in our paper are different in different regions and the parameter before the operator *curl* don't maintain sign in some regions, so the system is more complex and then the usual multigrid methods for Maxwell problem won't work.

In this paper, we will propose an iterative two-grid method for the edge finite element discretizations (a saddle-point system) of PML equations for a Maxwell scattering problem in two dimensions. Unlike the traditional two-grid method for elliptic problems, we need to take care of the kernel of operator *curl*. In detail, we first use a fine space to solve a discrete saddle-point system of $H(grad)$ variational problems, denoted by auxiliary system 1. Secondly, we use a coarse space to solve the original saddle-point system. At last, we use a fine space again to solve a discrete $\boldsymbol{H}(curl)$-elliptic variational problems, denoted by auxiliary system 2. Furthermore, we design a regularization diagonal block preconditioner for auxiliary system 1 since its algebraic system is still an indefinite saddle-point system. In view of the algebraic system of auxiliary system 2 it is a diagonal block matrix with each diagonal elements is a $\boldsymbol{H}(curl)$-elliptic operator, we choose PCG method based on $H$-$X$ preconditioner [14] as a solver. Numerical experiments show that the above two preconditioners are efficient and stable. With this method, the solution of the original problem in a fine grid is