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Application of Improved (G'/G)–Expansion Method to Traveling Wave Solutions of Two Nonlinear Evolution Equations

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Abstract. In this work, the improved (G'/G)-expansion method is proposed for constructing more general exact solutions of nonlinear evolution equation with the aid of symbolic computation. In order to illustrate the validity of the method we choose the RLW equation and SRLW equation. As a result, many new and more general exact solutions have been obtained for the equations. We will compare our solutions with those gained by the other authors.

AMS subject classifications: 35C07, 35C08, 35A25

Key words: RLW equation, SRLW equation, improved (G'/G)-expansion method, traveling wave solution.

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1 Introduction

Nonlinear evolution equations (NLEEs) have been the subject of study in various branches of mathematical physical sciences such as physics, biology, chemistry. The investigation of traveling wave solutions to nonlinear evolution equations play an important role in the study of nonlinear physical phenomena.

Many effective methods [1–10] have been presented such as, inverse scattering transform method [1], Hirota's method [2], variational iteration method [3], the homogeneous balance method [4], Backlund and Darboux transformation method [5], the sine-cosine function method [6], the Jacobi elliptic function method [7], auxiliary equation method [8–10] and others.

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Recently, the (G'/G)-expansion method, firstly introduced by wang et al. [11] has become widely used to search for various exact solutions of NLEEs [12–16]. The value of the (G'/G)-expansion method is that one treats nonlinear problems by essentially linear methods. Very lately to enhance the (G'/G)-expansion method and expand the range of its applicability, further research has been carried out by several authors. Zhang et al. [14] improved the method to deal with the (2+1)-dimensional Broer-Kaup equation with variable coefficients. Shehata [15] modified the method to derive traveling wave solutions for nonlinear Schrodinger equation and the cubic-quintic Ginzburg Landau equation. Zhang [17] explored a new application of this method to some special nonlinear evolution equations, the balance numbers of which are not positive integers.

(G'/G)-expansion method and the transformed rational function method used by W. X. Ma [18, 19] have a common idea. That is, we firstly put the given NLEE into the corresponding ordinary differential equation (ODE), then the ODE can be transformed into a systems of of algebraic polynomials with the determining constants. By the solutions of the ODE, we can obtain the exact traveling solutions and rational solution of the NLEE. However, to get the N-soliton and N-wave solution of the PDE, we may consider the linear superposition principle [20] and multiple exp-function method [21], these methods can be applied to the Hirota bilinear equations and others. Ma [20, 21] have obtained many N-wave solutions of the (3+1)-dimensional potential-Yu-Toda-Sasa-Fukuyama equation, the (3+1)-dimensional KP equations et al. There is on application of (G'/G)-expansion method in this area so far.

In this paper, we will propose the improved (G'/G)-expansion method to construct more general exact solutions of nonlinear evolution equations (NLEES). For illustration, we restrict our attention to the study of Regularized long wave (RLW) equation and Symmetric RLW equation and successfully construct many new and more general exact solutions.

2 Description of the improved (G'/G)-expansion method

Suppose that we have a NLEE for $u(x,t)$ in the form

$$P(u, u_x, u_t, u_{xt}, u_{xx}, \cdots) = 0, \tag{2.1}$$

where P is a polynomial in its arguments, which includes nonlinear terms and the highest order derivatives. The transformation $u(x,t) = u(\xi)$, $\xi = x - \omega t$ reduces Eq. (2.1) to the ordinary differential equation (ODE)

$$H(u, u_\xi, u_{\xi\xi}, u_{\xi\xi\xi}, \cdots) = 0. \tag{2.2}$$

By virtue of the extended tanh-function method, we assume that the solution of Eq. (2.2) is of the form

$$u(\xi) = \sum_{i=0}^{l} a_i F^i(\xi), \tag{2.3}$$