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Nonlinear Oscillations Analysis of the Elevator Cable in a Drum Drive Elevator System

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Abstract. This paper aims to investigate nonlinear oscillations of an elevator cable in a drum drive. The governing equation of motion of the objective system is developed by virtue of Lagrangian's method. A complicated term is broached in the governing equation of the motion of the system owing to existence of multiplication of a quadratic function of velocity with a sinusoidal function of displacement in the kinetic energy of the system. The obtained equation is an example of a well-known category of nonlinear oscillators, namely, non-natural systems. Due to the complex terms in the governing equation, perturbation methods cannot directly extract any closed form expressions for the natural frequency. Unavoidably, different non-perturbative approaches are employed to solve the problem and to elicit a closed-form expression for the natural frequency. Energy balance method, modified energy balance method and variational approach are utilized for frequency analyzing of the system. Frequency-amplitude relationships are analytically obtained for nonlinear vibration of the elevator's drum. In order to examine accuracy of the obtained results, exact solutions are numerically obtained and then compared with those obtained from approximate closed-form solutions for several cases. In a parametric study for different nonlinear parameters, variation of the natural frequencies against the initial amplitude is investigated. Accuracy of the three different approaches is then discussed for both small and large amplitudes of the oscillations.

AMS subject classifications: 70K30, 70E55

Key words: Nonlinear oscillation, perturbation methods, non-perturbative approach, frequency-amplitude relationship.

1 Introduction

The nonlinearity and complexity of the real world phenomena enforce leading edge scientists to develop innovative ways for understanding of the enigmatic behavior of the

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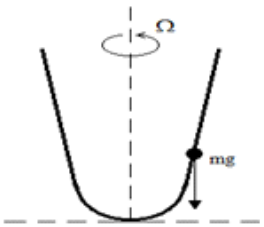
nature. There is a broad compendium of detailed information and knowledge regarding nonlinear systems in different outstanding books. Inevitably, a number of researchers have devoted their time and effort to find potent approaches for investigating of the nonlinear phenomena. Owing to the difficulty of the corresponding governing equations, a plenty of approaches have been developed so far to solve such sophisticated problems. As the earliest effort in the mysterious and wondrous road of the investigation of the nonlinear phenomena, perturbation-based methods can be referred [1,2]. The abovementioned types of straightforward methods have been widely used and modified mainly by Nayfeh et al. [1–3] and recently by other researchers [4–12]. They have been also employed and ameliorated for analyzing of diverse types of nonlinear structural systems. Adomian Decomposition method [16], Variational iteration method [17] and Homotopy analysis method [18–20] have been recently broached by researchers to solve nonlinear systems. In comparison with perturbation methods, more profound knowledge and perception can be provided using the aforesaid new analytical methods [21–26]. Hamiltonian approach [27], Energy Balance Method [28], Modified Energy Balance Method [29], Max-Min approach [30], Variational approach [31] and frequency amplitude formulation [32] can be cited as newly developed non-perturbative approaches. Efficiency and potency of these analytical methods have been proved in variety of complicated nonlinear cases, namely, non-natural systems, non-Hamiltonian systems, fractional order systems and generalized Duffing systems [33–42]. Many different kinds of non-natural systems have been analyzed so far by several researchers [8, 43–50]. The following Examples 1.1-1.6 (with models and governing equations) of non-natural systems succinctly delineates a plenty of well-known non-natural systems which have been analytically investigated so far. In the present study, three different non-perturabtive approaches are employed to extract frequency-amplitude relationship for nonlinear oscillations of an elevator cable in a drum drive elevator system. The governing equation of the nonlinear system is developed using Lagrangian's method. In accordance with the obtained ODE, it can be mentioned that the considered system is a non-natural structure.

Example 1.1. Motion of a particle on rotating parabola [8, 43, 44].



The governing equation is

$$(1+4p^2x^2)\ddot{x}+\Lambda x+4p^2\dot{x}^2x=0.$$

The motion of the system described by the following parabola in which p is a positive