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Solution of Two-Dimensional Stokes Flow Problems Using Improved Singular Boundary Method

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Abstract. In this paper, an improved singular boundary method (SBM), viewed as one kind of modified method of fundamental solution (MFS), is firstly applied for the numerical analysis of two-dimensional (2D) Stokes flow problems. The key issue of the SBM is the determination of the origin intensity factor used to remove the singularity of the fundamental solution and its derivatives. The new contribution of this study is that the origin intensity factors for the velocity, traction and pressure are derived, and based on that, the SBM formulations for 2D Stokes flow problems are presented. Several examples are provided to verify the correctness and robustness of the presented method. The numerical results clearly demonstrate the potentials of the present SBM for solving 2D Stokes flow problems.

AMS subject classifications: 76D07, 76M25

Key words: Singular boundary method, origin intensity factor, Stokes flow, fundamental solution.

1 Introduction

The incompressible viscous flow in slow motion, known as Stokes flow, is a classical problem in fluid dynamics. It can be regarded as a subset of Navier-Stokes flows, and has been widely applied in the industry. For the numerical analysis of Stokes flow problem, three formulations are well-known: vorticity-stream vector function, vorticity-velocity approach and primitive variable (velocity-pressure) approach.

Many numerical methods have been applied to the solution of the Stokes equations, such as finite difference method (FDM) [1], finite element method (FEM) [2] and boundary element method (BEM) [3]. More recently, with the development of the meshless

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method, some other simpler and fast methods have appeared in the literature, particularly the method of fundamental solution (MFS) which has been used to solving Stokes flow problems [4–6]. Over the last decade, the main advantage of the MFS, namely the simple computational implementation, has been recognized. However, a fictitious boundary for the distribution of source points is required in the MFS, which enormously restricts its application in the practical project.

Some efforts have been put in the elimination of this drawback, and numerous techniques are developed. A comprehensive review with respect to these methods can be found in the [7–9]. In this work, we focus on the singular boundary method (SBM), recently proposed by Chen and his collaborators [8, 10, 11]. For the SBM, the source and collocation points are both distributed on the physical boundary, thus there is no need of fictitious boundary required in the MFS. But, the singularity of the fundamental solution is caused when the source point coincides with the collocation point. To isolate the singularity of the fundamental solution, the origin intensity factor is introduced, and thus how to determine the factor is the key issue of the SBM. Prior to this study, the SBM has since been successfully applied to potential [8] and elasticity problems [11], in which we find that the method has a well performance.

The object of this paper is to extend the SBM for solving Stokes flow problems. Different from our previous SBM solution of 2D elasticity problems [11], a new regularization technique can accurately remove the singularities of the fundamental solution and its derivatives, and consequently, the origin intensity factors can be determined directly without requiring sample nodes as in [11]. Furthermore, based on the corresponding origin intensity factors, the SBM formulations for the velocity, traction and pressure are established. The rest of this paper is organized as follows. In Section 2, the governing equation of the Stokes problems is introduced, and the origin intensity factors for the velocity, traction and pressure are derived. Section 3 provides four numerical examples to assess the performance of the proposed SBM scheme. Finally, some conclusions are provided in Section 4.

2 The SBM formulation for Stokes problems

In this paper, we always assume that Ω is a bounded domain in R^2, Ω^e is its open complement; $\Gamma=\partial\Omega$ denotes their common boundary; $\mathbf{n}(\mathbf{x})$ and $\mathbf{t}(\mathbf{x})$ are the unit outward normal vector and tangential vector of Γ to domain Ω at point $\mathbf{x}$, respectively.

Assuming the incompressibility for the fluid medium, the governing equation for the steady-state Stokes flow problems can be expressed as follows:

$$\text{Momentum equation:} \qquad -p_{,i}+\mu u_{i,jj}=0, \quad \mathbf{x}\in\Omega, \tag{2.1a}$$

$$\text{Continuity equation:} \qquad u_{i,i}=0, \quad \mathbf{x}\in\Omega, \tag{2.1b}$$

$$\text{Boundary condition:} \qquad \begin{cases} u_i=\bar{u}_i, & \mathbf{x}\in\Gamma_u, \\ t_i=\bar{t}_i, & \mathbf{x}\in\Gamma_t, \end{cases} \tag{2.1c}$$