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A Consistent Characteristic Boundary Condition for General Fluid Mixture and Its Implementation in a Preconditioning Scheme

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Abstract. Characteristic boundary conditions that are capable of handling general fluid mixtures flow at all flow speeds are developed. The formulation is based on fundamental thermodynamics theories incorporated into an efficient preconditioning scheme in a unified manner. Local one-dimensional inviscid (LODI) relations compatible to the preconditioning system are proposed to obtain information carried by incoming characteristic waves at boundaries accurately. The approach has been validated against a variety of sample problems at a broad range of fluid states and flow speeds. Both acoustic waves and hydrodynamic flow features can pass through the boundaries of computational domain transparently without any unphysical reflection or spurious distortion. The approach can be reliably applied to fluid flows at extensive thermodynamic states and flow speeds in numerical simulations. Moreover, the use of the boundary condition shows to improve the computational efficiency.

AMS subject classifications: 65M10, 78A48

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1 Introduction

Proper implementation of boundary conditions is of great importance in obtaining reliable and accurate numerical solutions of compressible flows. Much effort has been

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applied to develop robust numerical boundary conditions based on the method of characteristics (MOC) [1]. The technology was originated from the classical characteristic solution of differential equations of a hyperbolic system [2]. The MOC method was later extended to address multidimensional inviscid system with non-reflective acoustic boundary conditions [3]. Poinsot and Lele [4] further developed an approach capable of treating viscous compressible flows. They derived local one-dimensional inviscid (LODI) relations to calculate quantities associated with incoming characteristic waves and improved the accuracy of the method. The viscous terms are added separately and can be relaxed smoothly to inviscid flows as viscous effects diminish. Except for problems with gently varying flow and weak acoustic wave near boundaries, the boundary conditions based on characteristic method can usually get more accurate and physical results than simple extrapolation method or simplified Riemann invariants boundary conditions. In addition, the method gives rise to a more robust solution procedure. Baum et al. [5] extended the characteristic boundary conditions from equations which only can consider perfect gas with constant homogeneous thermodynamic properties to accommodate multi-component reactive flows with variable thermodynamic properties. Although the characteristic boundary conditions have been widely used, most of those studies, however, focused on treating the fluid flows in the ideal gas regime. Very limited work has been conducted to develop a consistent boundary treatment for fluid flows under high-pressure and low-temperature conditions, for which the method based on ideal gas does not work and real fluids effects must be considered [6].

Many fluid flows involve thermodynamic states in the trans-critical and supercritical regimes, where the real-fluid effect is notably strong. Fluid properties may experience severe variations when approaching the liquid-gas critical point [7] (see Fig. 3). This is important in heat transfer and fluid dynamics and raises a challenge to the boundary condition treatment. For example, a very small thermal disturbance can generate enormous unsteadiness in the form of shock and expansion waves in a near-critical fluid due to the abnormally high fluid compressibility, a phenomenon known as the piston-effect in near-critical fluids [7]. The boundary conditions must be treated properly to avoid unphysical oscillations and reflections that will then propagate into the computational domain and ruin the solution. Okong'o and Bellan [8] extended MOC based boundary conditions from ideal gas to multi-component real-fluid mixtures. The formulations have been validated against the convection of a single vortex in supercritical heptane/nitrogen homogeneous mixture with supersonic flow speeds. Although it has been demonstrated that the extended characteristic boundary condition can successfully treat fluid flows with moderate real-fluid effects in high-speed flow regimes, the method cannot be directly implemented to handle transcritical or supercritical fluid flows at very low flow speeds because there is no low Mach number flow treatment technique such as the preconditioning method [9–11].

In this paper, we aim at developing consistent boundary conditions for general fluid mixtures flow at all speeds. The acoustic wave propagation related formulations, such as system eigenvalues and eigenvectors, are derived based on the unified