Advances in Applied Mathematics and Mechanics
Adv. Appl. Math. Mech., Vol. **10**, No. 1, pp. 242-260

DOI: 10.4208/aamm.OA-2017-0038
February 2018

A New Explicit Symplectic Fourier Pseudospectral Method for Klein-Gordon-Schrödinger Equation

Yanhong Yang[1,2], Yongzhong Song[1], Haochen Li[3] and Yushun Wang[1,*]

[1] *Jiangsu Key Laboratory for NSLSCS, Institute of Mathematics, School of Mathematical Sciences, Nanjing Normal University, Nanjing 210023, China*
[2] *Department of Mathematics, College of Taizhou, Nanjing Normal University, Taizhou 225300, China*
[3] *LMAM, CAPT and School of Mathematical Sciences, Peking University, Beijing 100871, China*

Received 14 February 2017; Accepted (in revised version) 15 June 2017

Abstract. In this paper, we propose an explicit symplectic Fourier pseudospectral method for solving the Klein-Gordon-Schrödinger equation. The key idea is to rewrite the equation as an infinite-dimensional Hamiltonian system and discrete the system by using Fourier pseudospectral method in space and symplectic Euler method in time. After composing two different symplectic Euler methods for the ODEs resulted from semi-discretization in space, we get a new explicit scheme for the target equation which is of second order in space and spectral accuracy in time. The canonical Hamiltonian form of the resulted ODEs is presented and the new derived scheme is proved strictly to be symplectic. The new scheme is totally explicit whereas symplectic scheme are generally implicit or semi-implicit. Linear stability analysis is carried and a necessary Courant-Friedrichs-Lewy condition is given. The numerical results are reported to test the accuracy and efficiency of the proposed method in long-term computing.

AMS subject classifications: 65M70, 65M12, 65M06
Key words: Klein-Gordon-Schrödinger equation, Fourier pseudospectral method, symplectic scheme, explicit scheme.

1 Introduction

The Klein-Gordon-Schrödinger (KGS) equation

$$\begin{cases} i\varphi_t+\dfrac{1}{2}\varphi_{xx}+u\varphi=0, & x\in\Omega=[x_L,x_R], \quad t>0, \\ u_{tt}-u_{xx}+u-|\varphi|^2=0, & x\in\Omega=[x_L,x_R], \quad t>0, \end{cases} \tag{1.1}$$

*Corresponding author.
Emails: yzynj@163.com (Y. H. Yang), yzsong@njnu.edu.cn (Y. Z. Song), wangyushun@njnu.edu.cn (Y. S. Wang)

is a classical model used to describe a system of conserved scalar nucleons interacting with neutral scalar meson coupled through the Yukawa interaction in [6], where the complex-valued function $\varphi=\varphi(x,t)$ represents a scalar nucleon field, the real-valued function $u=u(x,t)$ represents a scalar meson field and $i=\sqrt{-1}$ (also see [14]).

Let $\varphi(x,t)=q(x,t)+ip(x,t)$, where $q(x,t)$ and $p(x,t)$ are real functions and let $u_t(x,t)=2v(x,t)$. The KGS equation (1.1) can be written as

$$\begin{cases} p_t=\dfrac{1}{2}q_{xx}+uq, \\ q_t=-\dfrac{1}{2}p_{xx}-up, \\ v_t=-\dfrac{1}{2}(u_{xx}-u+p^2+q^2), \\ u_t=2v. \end{cases} \tag{1.2}$$

In this paper, we consider the periodic boundary conditions

$$p(x+L,t)=p(x,t),\quad q(x+L,t)=q(x,t),\quad u(x+L,t)=u(x,t),\quad v(x+L,t)=v(x,t),$$

and initial conditions

$$p(x,0)=p_0(x),\quad q(x,0)=q_0(x),\quad u(x,0)=u_0(x),\quad v(x,0)=v_0(x),$$

where $L=x_R-x_L$. Let $z=(p,v,q,u)^T$. Then (1.2) can be written as an infinite-dimensional Hamiltonian system

$$\frac{dz}{dt}=J^{-1}\frac{\delta H(z)}{\delta z} \tag{1.3}$$

with

$$J=\begin{pmatrix} 0 & I_2 \\ -I_2 & 0 \end{pmatrix},$$

I_2 is the identity matrix of dimension 2 and the Hamiltonian

$$H(z)=\int_{x_L}^{x_R}\left[-\frac{1}{4}(p_x^2+q_x^2)+\frac{1}{4}u^2-\frac{1}{4}u_x^2+v^2-\frac{1}{2}u(p^2+q^2)\right]dx.$$

The KGS equation posses the mass conservation law

$$\mathcal{M}(t)=\int_{x_L}^{x_R}|\varphi|^2dx=\mathcal{M}(0)$$

and the energy conservation law

$$\mathcal{H}(t)=\int_{x_L}^{x_R}\frac{1}{4}(u^2+u_t^2+u_x^2+|\varphi_x|^2)-\frac{1}{2}u|\varphi|^2dx=\mathcal{H}(0).$$