Advances in Applied Mathematics and Mechanics
Adv. Appl. Math. Mech., Vol. **10**, No. 1, pp. 159-183

DOI: 10.4208/aamm.OA-2017-0020
February 2018

Nonlinear Dynamics of Clamped Initial Imperfect Functionally Graded Material Circular Cylindrical Shell Considering the Axisymmetric Mode

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Received 26 January 2017; Accepted (in revised version) 18 May 2017

Abstract. This paper investigates the dynamic responses of clamped-clamped functionally graded material circular cylindrical shell at both ends with small initial geometric imperfection and subjected to complex loads. The small initial geometric imperfection of the cylindrical shell is characterized with the shape of hyperbolic function. The effects of radial harmonic excitation combined with thermal loads are considered. The classical theory and von-Karman type nonlinear geometric equation are applied to obtain partial differential equation of the functionally gradient material circular cylindrical shell by the Hamilton's principle. The partial differential dynamic equations are truncated by the Galerkin technique, using the modal expansion with the inclusion of axisymmetric and asymmetric modes. The effective material properties vary in the radial direction following a power-law distribution accordance with the volume fractions. The effects of volume fraction indexes, ratios of thickness-radius and length-radius on the first three dimensionless natural frequencies of the perfect cylindrical shell and its counterpart with imperfection are given. The effects of radial external loads, initial geometric imperfections and volume fraction index on the nonlinear dynamic response of the clamped-clamped FGM circular cylindrical shell are discussed by numerical calculation.

AMS subject classifications: 34A34, 34K18

Key words: Vibration, circular cylindrical shell, functionally graded materials, geometric imperfections, clamped-clamped.

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1 Introduction

The functionally gradient material (FGM) circular cylindrical shell structures whose material properties vary gradually and smoothly in thickness direction have been found to be widely applied in space vehicles, nuclear plants and so on due to their capacity to withstand high levels of thermal, mechanical and acoustic pressure. Under complicated working conditions these shells may cause a complicated nonlinear dynamics [1].

Recently, the dynamics of FGM circular cylindrical shells have attracted increasing research efforts. Firstly, the focus is put on the studies of natural frequencies of the FGM circular cylindrical shell. Some works dealt with the effect of physical and geometric parameters and edge conditions on the value of natural frequencies for functionally graded material circular cylindrical shell, see [2–5]. Assumed the axisymmetric conditions, Asgari and Akhlaghi [6] presented natural frequencies of thick hollow limited length FGM cylinders, according to 3D equations of elasticity.

Bhangale and Ganesan [7] analyzed the frequency behavior of vibrating non-homogeneous FGM magneto-electro-elastic finite cylindrical shells with simply supported boundary (SS). The influence of magnetic and piezoelectric on the frequency of structures is evaluated. The free vibration of functionally graded material cylindrical shells was studied by Cao and Wang [8]. There are some holes in the FGM cylindrical shells. The effects of holes shape, number and location on the frequencies were analyzed. Sepiani et al. [9] researched the buckling and free vibration properties of cylindrical shell with static and harmonic excitation along axial direction. The inner of the shell is the FGM layer and the out surface of the circular cylindrical shell is isotropic elastic layer. Both the first order shear deformation theory and the classical shell theory were used in theoretical formulations.

Using the generalized thermo-elasticity theory and second-order shear deformation shell theory, Bahtui and Eslami [10] researched the response of FGM circular cylindrical shell which is subjected to thermal shock load. Considering the effect of thermal and mechanical coupling combined with rotary inertia, Galerkin method and the Laplace transform were used to formulate the problem. Ng et al. [11] studied the influences of the volume fraction on the parametric response of FGM cylindrical shells under harmonic axial excitation, especially the positions and range of stability.

About the nonlinear dynamics, Mahmoudkhani et al. [12] used Donnell's nonlinear shallow shell theory and multiple scales methods to study the periodic solutions and their stability of simply supported circular cylindrical shells with the primary resonance. The sufficient and necessary conditions for the emergence of companion mode were discussed. Bichetal and Nguyen [13] investigated nonlinear amplitude frequency response and nonlinear dynamic of FGM circular cylindrical shells by using Donnell shell theory. Hao et al. [14] presented the nonlinear dynamics of SS functionally graded material circular cylindrical shell with 1:2 internal resonance between first two modes but without considering the axisymmetric mode. On the condition that primary resonance and 1:2 internal resonance of the system between two modes, Du and Li [15] dealt with the non-